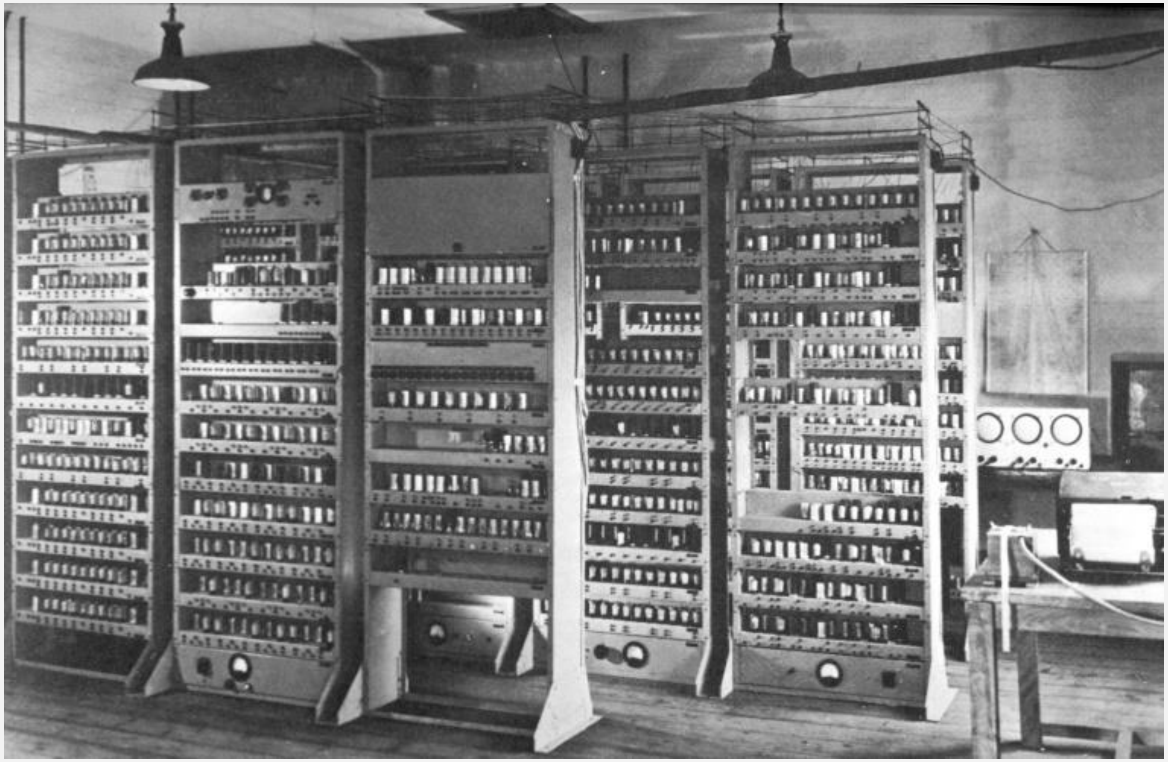
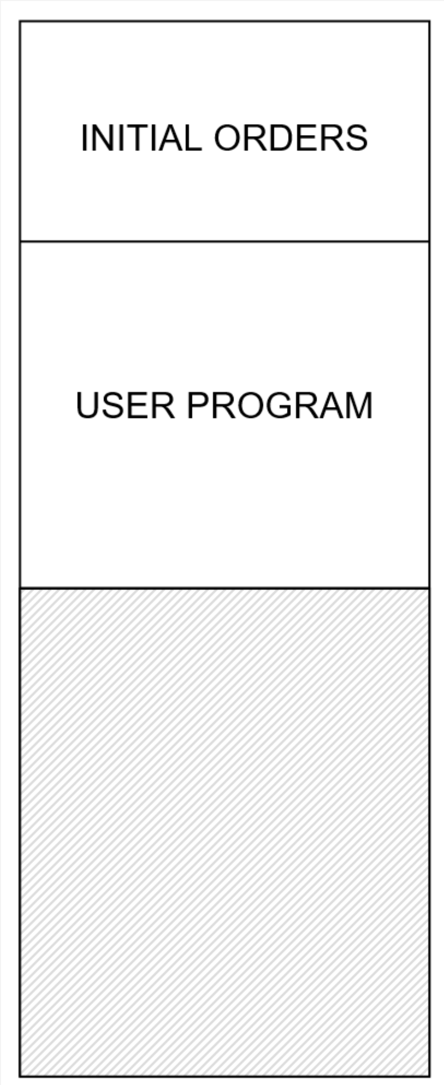
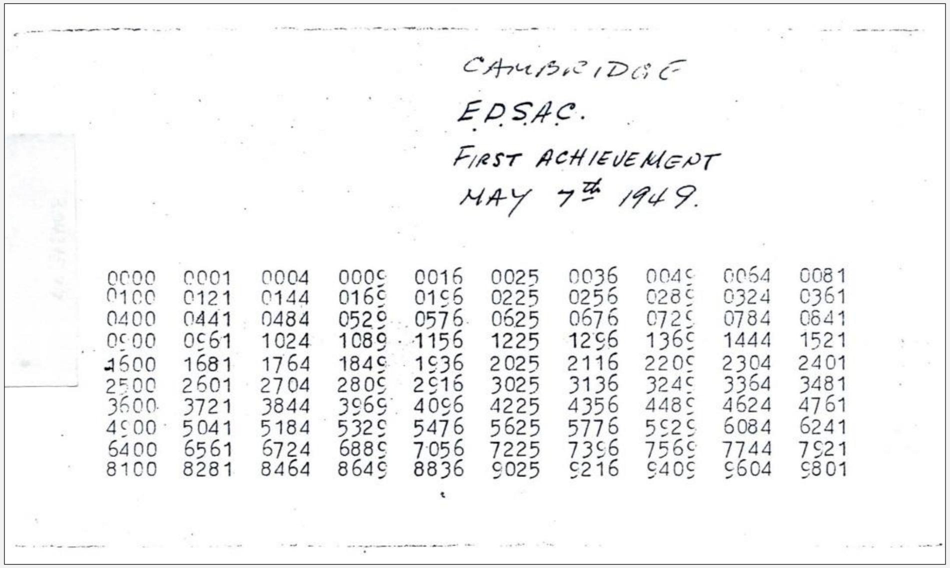
EDSAC PROGRAMMING AND APPLICATIONS



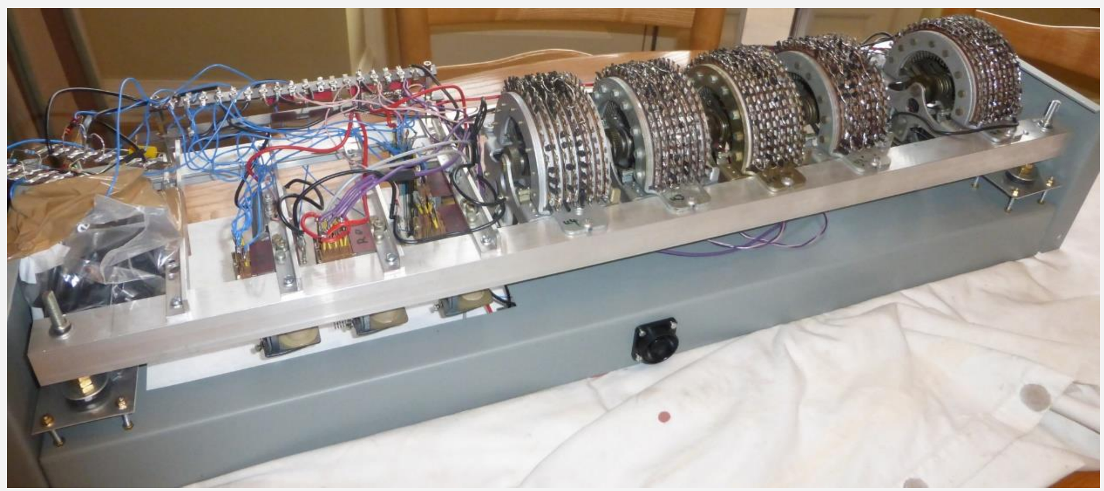
EDSAC springs to life, 6 May 1949





CAMBRIDGE
EDSAC.
FIRST ACHIEVEMENT
MAY 7th 1949.

0000	0001	0004	0009	0016	0025	0036	0049	0064	0081
0100	0121	0144	0169	0196	0225	0256	0289	0324	0361
0400	0441	0484	0529	0576	0625	0676	0729	0784	0841
0900	0961	1024	1089	1156	1225	1296	1369	1444	1521
1600	1681	1764	1849	1936	2025	2116	2209	2304	2401
2500	2601	2704	2809	2916	3025	3136	3249	3364	3481
3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
6400	6561	6724	6889	7056	7225	7396	7569	7744	7921
8100	8281	8464	8649	8836	9025	9216	9409	9604	9801



David Wheeler’s Initial Orders – wired on to uniselectors

UNIVERSITY OF CAMBRIDGE
COMPUTER LABORATORY

Head of Department
Prof. M. V. Wilkes, F.R.S.

Director of the University
Computing Service
Dr. D. F. Hartley

Corn Exchange Street
Cambridge CB2 3QG

Telephone

Please note that our new telephone number is Cambridge (0223) 352435

2 August 1979

Mr M. Campbell-Kelly
Sunderland Polytechnic
Department of Mathematics
and Computer Studies
Priestman Building
Green Terrace
Sunderland SR1 3SD

Dear Mr Campbell-Kelly,

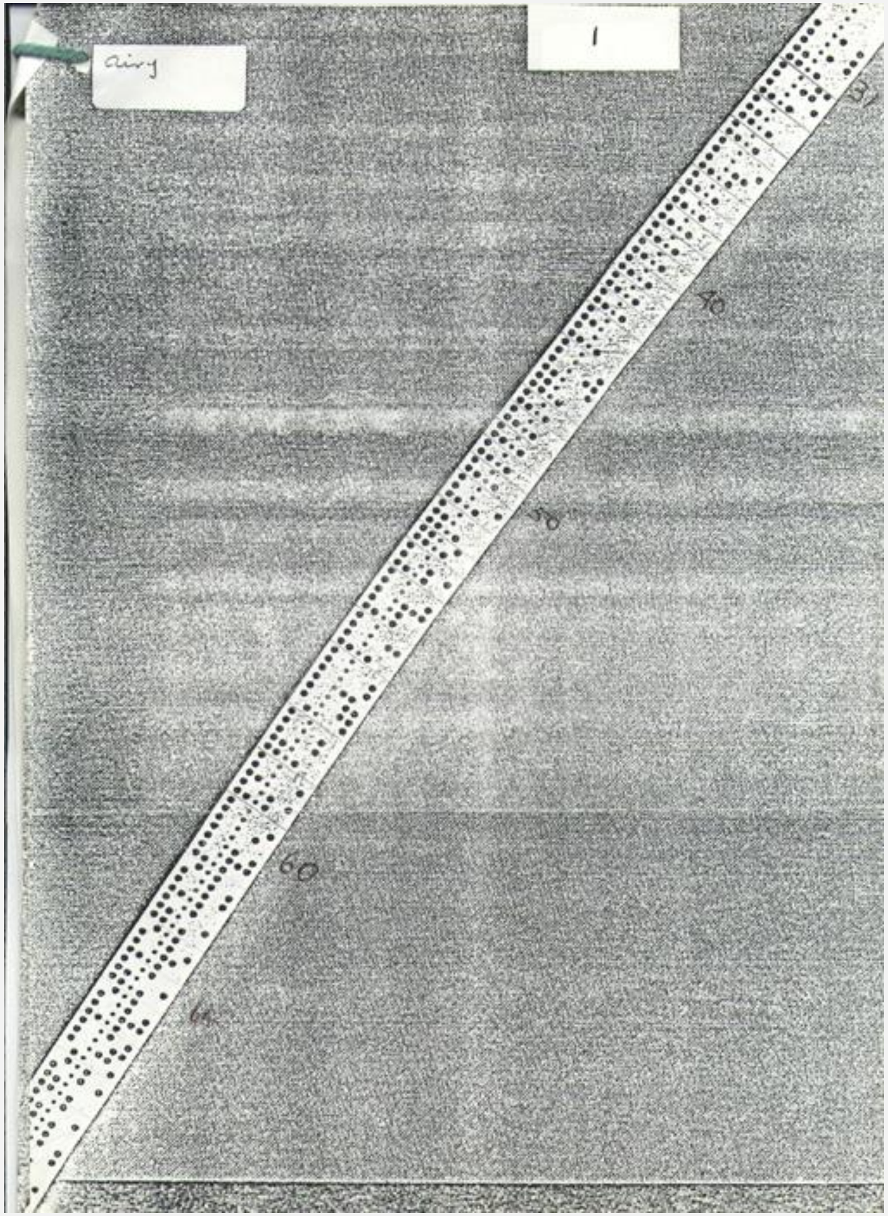
I thought that you might be interested in a very early EDSAC program tape that has come to light. The tape itself is in a rather fragile condition and I thought it best to make a Xerox copy. This I enclose.

The tape is labelled in pencil with the word 'Airy'. I also enclose a Xerox copy of the first page of a "Record of Programme Tapes" that I began to keep at the end of July 1949. Since the tape bears no number, it presumably belongs to just before this period. It is an early version of the program that was used to compute the table of the Airy integral that I published in 'Nature'.

Yours sincerely,

M. V. Wilkes

encs



Wilkes' Airy Program June-July 1949

cluding address was on "The Control of Population", by Dr. C. P. Blacker (secretary of the Eugenics Society). He dealt mainly with the manner in which the need to control population affects Great Britain and the Commonwealth, and how it will shortly present itself to the world. As regards Great Britain, he suggested that, in our present precarious economic conditions, our basic position would be safer if our population were reduced to within five or six per cent of the number we could feed from our own resources. In the case of the Commonwealth, he hoped that a Commonwealth Institute of Demographic Studies, concerned with collecting information, could be formed, and that its work could be supplemented by a Commonwealth Office of Population, designed to weld the Commonwealth into a demographic unity. In connexion with the world problem, he maintained that there is but one solution for countries like India and China, now struggling in the high stationary and early expanding phase of the demographic cycle—the deliberate control of fertility. Admitting that the task of transforming unbending forms of thought and custom, evolved over millennia and interwoven in the texture of religion, is formidable, there are signs that a sense of demographic realities is spreading among enlightened men in India and elsewhere. Under the impact of Government encouragement and feminist propaganda, and in the presence of specially provided facilities, what has been called the 'cake of custom' might break up more easily than seemed possible.

ELECTRONIC CALCULATING-MACHINE DEVELOPMENT IN CAMBRIDGE

By Dr. M. V. WILKES

In a recent article (see *Nature*, August 27, p. 341) an account was given of a conference on high-speed digital calculating machines held in the University Mathematical Laboratory, Cambridge. The article included a brief description of the EDSAC (electronic delay storage automatic calculator)[1,2], the large electronic calculating machine which has been built in the Laboratory, and its relationship to the various similar machines now under construction in England and the United States. It is intended to give here some further information about the EDSAC.

A digital calculating machine performs the operations of addition, subtraction, multiplication and division, and can, therefore, be used for solving any problem which can be reduced to arithmetical form. In a number of fields of research, mathematical formulation of the problems presented is possible, but analytical treatment of the resulting equations is either not feasible or is otherwise inappropriate. The methods of numerical analysis are, however, often applicable, and it may be expected that progress in such fields will be greatly accelerated by the application of high-speed calculating machines.

Much original work of a mathematical character may be necessary in order to reduce a problem to a form in which it can be put on the EDSAC. This will be specially true of problems which give rise to very large-scale computing operations such as become possible when a high-speed machine is available.

The photograph shown in Fig. 1 will give some idea of the size of the EDSAC. It contains about 3,000 valves and consumes 12 kilowatts of power. A problem is presented to the machine in the form of a punched paper tape of the kind used in telegraphy. The tape contains the programme, that is to say, instructions (in a coded form) for performing the successive arithmetical operations needed to solve the problem, and also any numerical data required. No other setting up of the machine besides putting the tape in the tape reader is necessary; the machine can, therefore, be switched rapidly from one problem to another. Fig. 2 is a photograph of the tape-reader with a tape in position. The results of the calculation are printed on a modified teleprinter. Instructions for the layout of the work and for printing any headings required must be included in the programme.

Except for the input and output mechanism, the EDSAC has no moving parts, all computing and control operations being performed by means of electronic circuits. Within the machine numbers are expressed in the scale of two and are represented by trains of pulses synchronized with a continuously running 'clock pulse' generator. If a pulse is present in a certain position in the train, the corresponding digit of the number is a 1; if there is no pulse, the digit is a 0.

Numbers expressed in this form are stored by a method depending on the use of an ultrasonic delay unit. The pulses are applied to a quartz crystal mounted at one end of a column of mercury, and give rise to ultrasonic pulses which travel through the mercury with the velocity of sound. On arrival at the far end they strike a second quartz crystal and are reconverted into electrical pulses. The time taken to traverse a column of mercury 5 ft. long is about 1 msec., and the interval between the beginning of one pulse and the beginning of the next is 2 μsec.; there can thus be as many as 500 pulses passing down the column at any one time. On emerging from the delay unit the pulses are amplified and reshaped, and passed back to the input of the delay unit. They then continue to circulate indefinitely and are available when required.

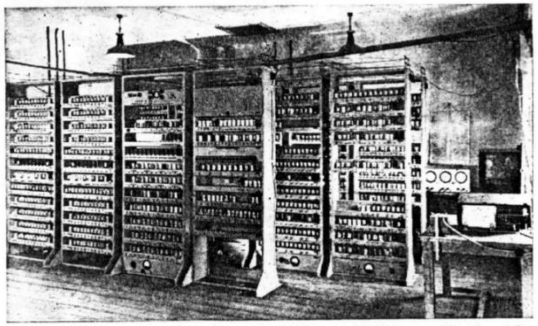
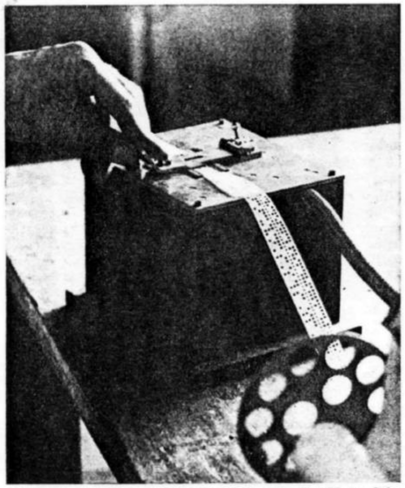


Fig. 1. A general view of the EDSAC. The racks in the front row contain (from left to right): part of the store (two racks), pulse generator, and input-output units. Behind are three racks containing the control, and, in the rear, the remainder of the store (two racks) and the arithmetical unit (three racks). On the extreme right of the photograph may be seen the tape-reader for the input tape, and the teleprinter on which results are printed



Fox Photos, Ltd.

Fig. 2. The tape-reader, with an input tape in position, and part of the teleprinter

Conversion of numbers to and from the scale of two is effected automatically during input and output by the machine itself, acting under the control of instructions included in the programme. No inconvenience is therefore caused to mathematical users by the fact that the machine works in an unfamiliar scale.

An example of one type of problem which can be solved by means of the EDSAC is the evaluation of Airy's integral $Ai(-x)$ by the numerical solution of the differential equation $y'' + xy = 0$. There are many different methods based on the use of finite-difference formulæ by which this can be done; the one which will be taken as an example depends on the use of the well-known central difference formula

$$\delta^2 y = (\delta x)^2 \left(y'' + \tfrac{1}{12}\,\delta^2 y''\right),$$

where δx is the interval of the argument. If this is expressed in terms of three adjacent values of y, namely, y_0, y_1 and y_2, corresponding to the three equally spaced values of x, that is, x_0, x_1 and x_2, and use made of the differential equation to eliminate y'', it follows that

$$y_2 = 2y_1 - y_0 - \frac{1}{12}(\delta x)^2 (x_0 y_0 + 10 x_1 y_1 + x_2 y_2).$$

If y_0 and y_1 are known, y_2 may be obtained from this equation, and by repeated application a solution of the differential equation may be traced out point by point. Since y_2 occurs on the right-hand side with a small coefficient, the equation may conveniently be solved by an iterative method.

The problem may now be said to be expressed in arithmetical form. In order to set out in further detail the operations to be performed by the machine, it will be assumed that the quantities y_0 and y_1 are held in the store of the machine in 'storage locations' numbered 100 and 101 respectively, and that storage location 102 contains a number η. η will change as the calculation proceeds, and will finally become equal to y_2; initially $\eta = y_1$. The various stages of the calculation are then as follow: (1) Evaluate

$$\eta' = 2y_1 - y_0 - \frac{1}{12}(\delta x)^2 (x_0 y_0 + 10 x_1 y_1 + x_2 y_2).$$

(2) Examine the sign of $|\eta' - \eta| - \varepsilon$, where ε is a small quantity specified in advance. If the sign is positive, replace η in storage location 102 by η' and repeat (1). If the sign is negative, proceed to (3). (3) Print y_0. (4) Replace y_0 in storage location 100 by y_1 from storage location 101, and y_1 in storage location 101 by y_2 from storage location 102 (η remains in storage location 102). Repeat (1).

In this way the machine proceeds to evaluate the function step by step, performing as many iterations as may be necessary each time.

The various additions, subtractions and other operations which go to make up the stages of the calculation given above must now be listed in detail and expressed in terms of the order code of the machine[3]. Suitable orders must be added for starting the process from specified initial conditions and for stopping it when the solution has proceeded sufficiently far. The orders, together with all numerical parameters required, are now punched on a paper tape. The tape is placed in the tape reader, and the machine proceeds to evaluate and print successive values of $Ai(-x)$ without further intervention from the operator.

Fig. 3 is a photograph of a table of $Ai(-x)$ at intervals of 0·05 computed and printed by the EDSAC. The argument has been written in by hand; if desired, the machine could be made to print the argument by including appropriate orders in the programme.

About 150 orders were required to compute and print the table shown, and the time taken was four minutes. Of this, only about twenty seconds was occupied in computing, the rest being taken up by input and printing. In many problems of a more complicated kind, the computing time might be expected to be a larger fraction of the whole.

x	Ai(-x)				
0·00	+.35503	+.36796	+.38085	+.39364	+.40628
	+.41872	+.43090	+.44276	+.45423	+.46524
	+.47573	+.48562	+.49485	+.50333	+.51100
	+.51777	+.52357	+.52833	+.53196	+.53439
1·00	+.53556	+.53539	+.53381	+.53077	+.52619
	+.52005	+.51227	+.50283	+.49170	+.47885
	+.46426	+.44793	+.42986	+.41008	+.38861
	+.36548	+.34076	+.31451	+.28680	+.25773
2·00	+.22741	+.19595	+.15348	+.13016	+.09615
	+.06160	+.02671	-.00834	-.04333	-.07807
	-.11233	-.14588	-.17850	-.20997	-.24004
	-.26849	-.29510	-.31964	-.34191	-.36169
3·00	-.37881	-.39310	-.40438	-.41254	-.41744
	-.41901	-.41718	-.41191	-.40319	-.39105
	-.37553	-.35674	-.33478	-.30981	-.28201
	-.25151	-.21886	-.18402	-.14742	-.10938
4·00	-.07027	-.03045	+.00968	+.04970	+.08921
	+.12778	+.16500	+.20044	+.23370	+.26440
	+.29215	+.31562	+.33750	+.35449	+.36737
	+.37593	+.38004	+.37959	+.37454	+.36490

Fig. 3. An example of work done by the machine. The function $Ai(-x)$ is tabulated for values of x from 0 to 4·95 at intervals of 0·05. Five consecutive values are given in each line

[1] Wilkes, *Proc. Roy. Soc.*, A, **195**, 274 (1948).
[2] Wilkes and Renwick, *Elec. Eng.*, **20**, 208 (1948).
[3] Wilkes, *J. Sci. Instr.*, **26**, 217 (1949).

Wilkes' Airy Program June-July 1949

27/7

~~27/1 A_1 x airys integral 101 — 15D with round off at order 80~~

27/2 A_2 corrected by putting E99S for E97S in orders 170, 175

27/3 A_3 Ai(−x) .05 to print 6D (rounded off)

28/7

A 4 Ai(−x) .05 with round off at order 80.

A 5. beginning of tape (as far as order 60, P120S7S) for Ai(−x) .05

A6. As A_2 but with Ai(.01) = .35761...

A 7 .05 with round off at 80 & dec round off to 5 figs (for Nature)

A 8 $\frac{d^2y}{dx^2} + xy = \frac{1}{\pi}$ interval .05.

A 9 A 8 corrected.

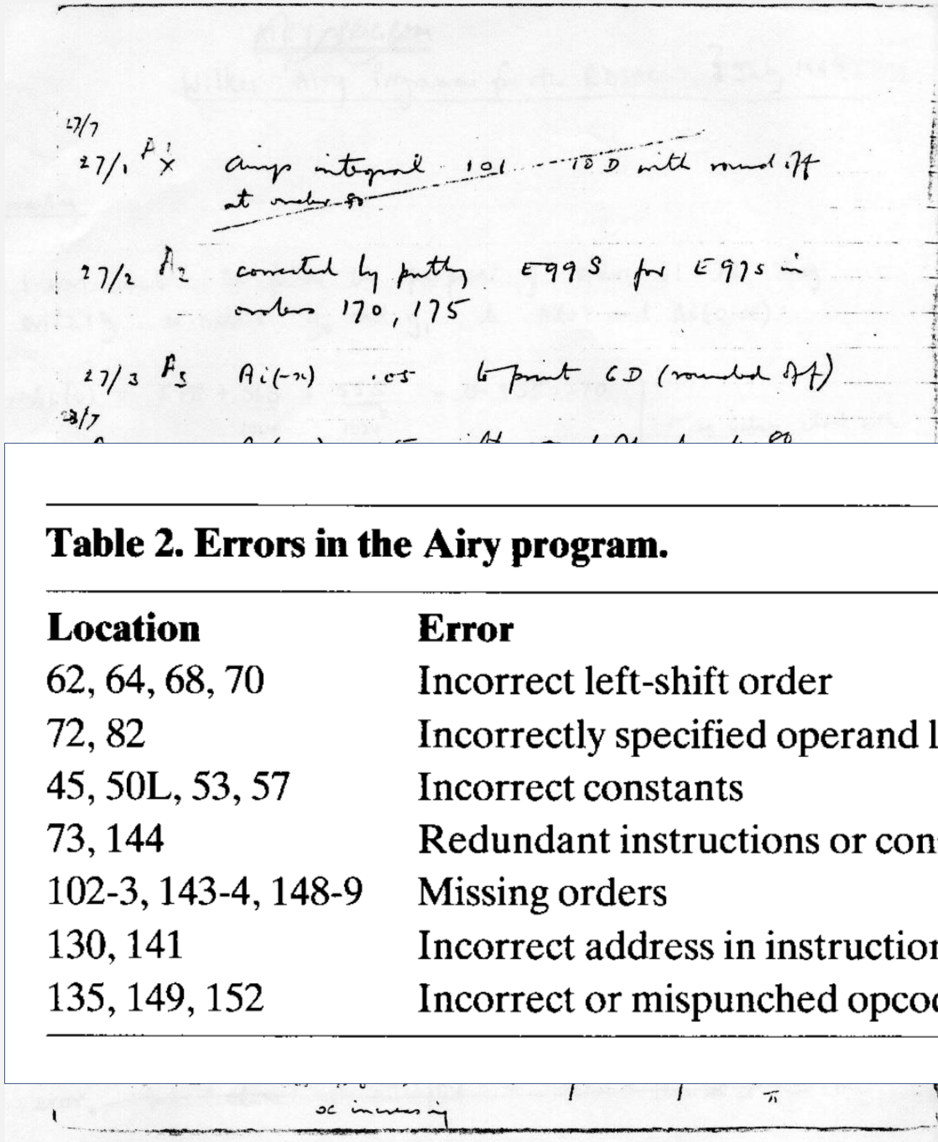
A 10 A 9 corrected. check.

W 11 As A2 but to print successive approx

12 ~~...~~ $y'' - xy = \frac{1}{\pi}$ at .01 x increasing

92	G 11~~2~~3 S	branch if $\lvert\eta'-\eta\rvert - e < 0$
3	T S	η' to η
4	A 42 L	
5	T 40 L	
6	E 7~~4~~3 S	repeat iteration
152 → 7	T S	x_1 to x_0 (all scaled by $2^{-11}.10$)
153 → 8	A 33 S	
9	T 34 S	
100	A 35 S	x_2 to x_1
1	U 33 S	
2	A 52 S	$x_2+\delta x$ to x_2 ?
3	T 35 S	
~~3~~4	A 38 L	y_1 to y_0
~~4~~5	T 36 L	
~~5~~6	A 42 L	y_2 to y_1
~~6~~7	T 38 L	
~~7~~8	H 119 S	$10x_1.2^{-11}.10$ to 44S
~~8~~9	V 33 S	
~~9~~110	L 4 S	
~~110~~1	T 44 S	
~~1~~2	E 74 S	repeat (3)
~~2~~3	T S	branch to print y_0
~~3~~4	E 127 S	

Wilkes' Airy Program June-July 1949



27/1 A_1' Airy integral 101 ... 10 D with round off at order ...

27/2 A_2 corrected by putting E 99 S for E 97 S in orders 170, 175

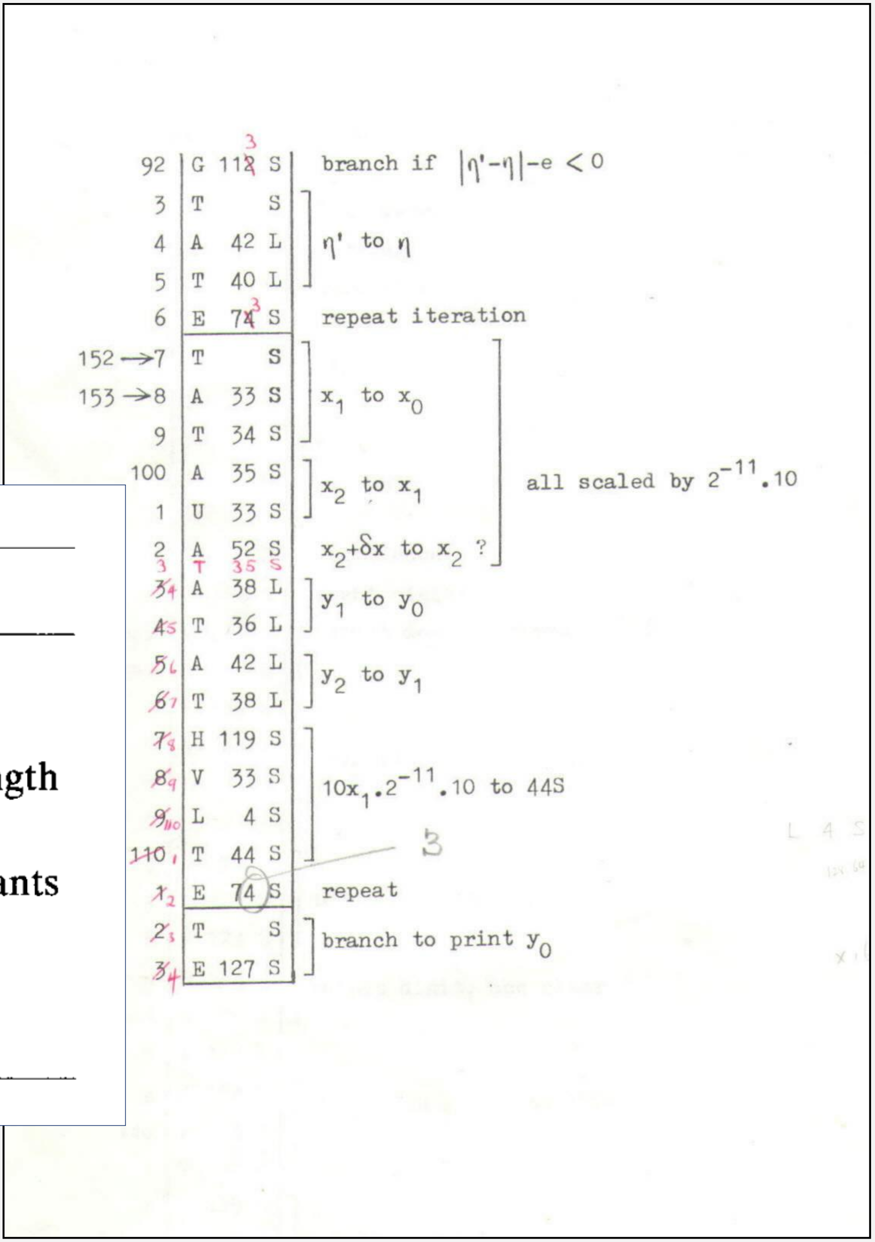
27/3 A_3 Ai(-x) .05 to print 6 D (rounded off)

28/7

x increasing

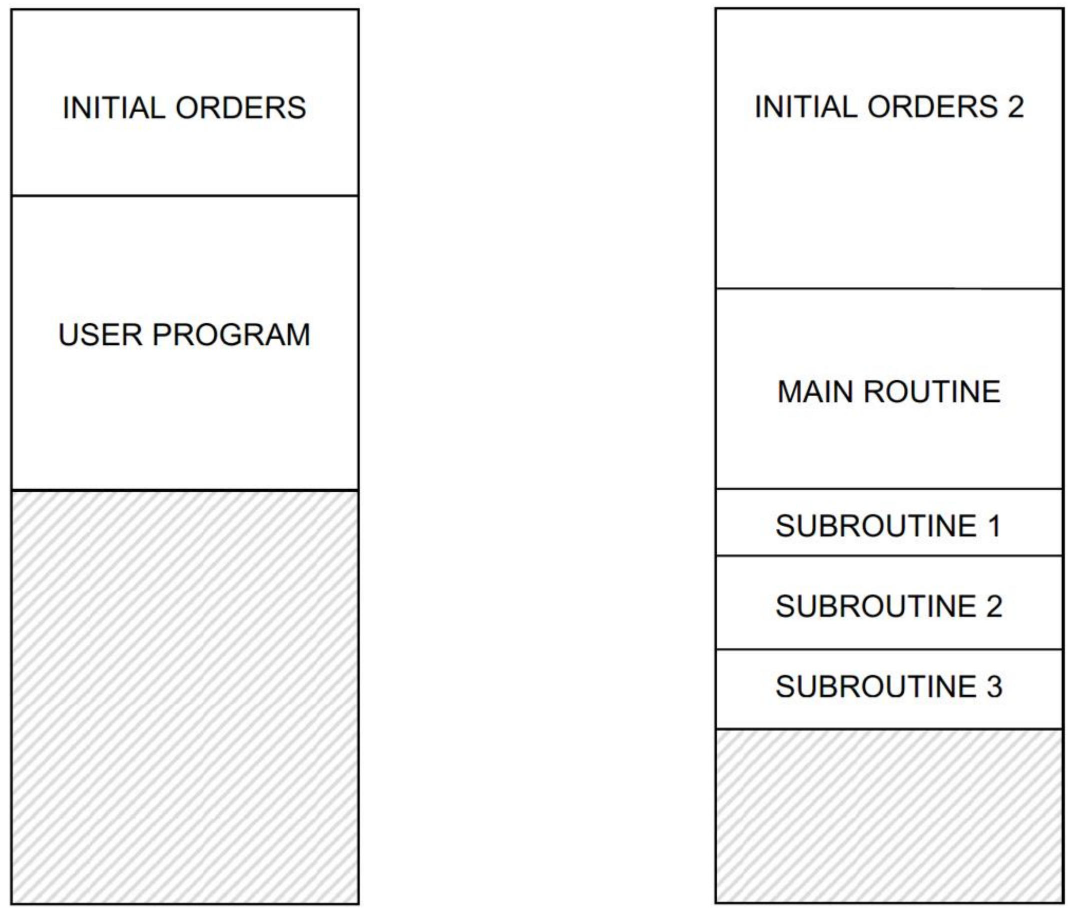
Table 2. Errors in the Airy program.

Location	Error
62, 64, 68, 70	Incorrect left-shift order
72, 82	Incorrectly specified operand length
45, 50L, 53, 57	Incorrect constants
73, 144	Redundant instructions or constants
102-3, 143-4, 148-9	Missing orders
130, 141	Incorrect address in instruction
135, 149, 152	Incorrect or mispunched opcode

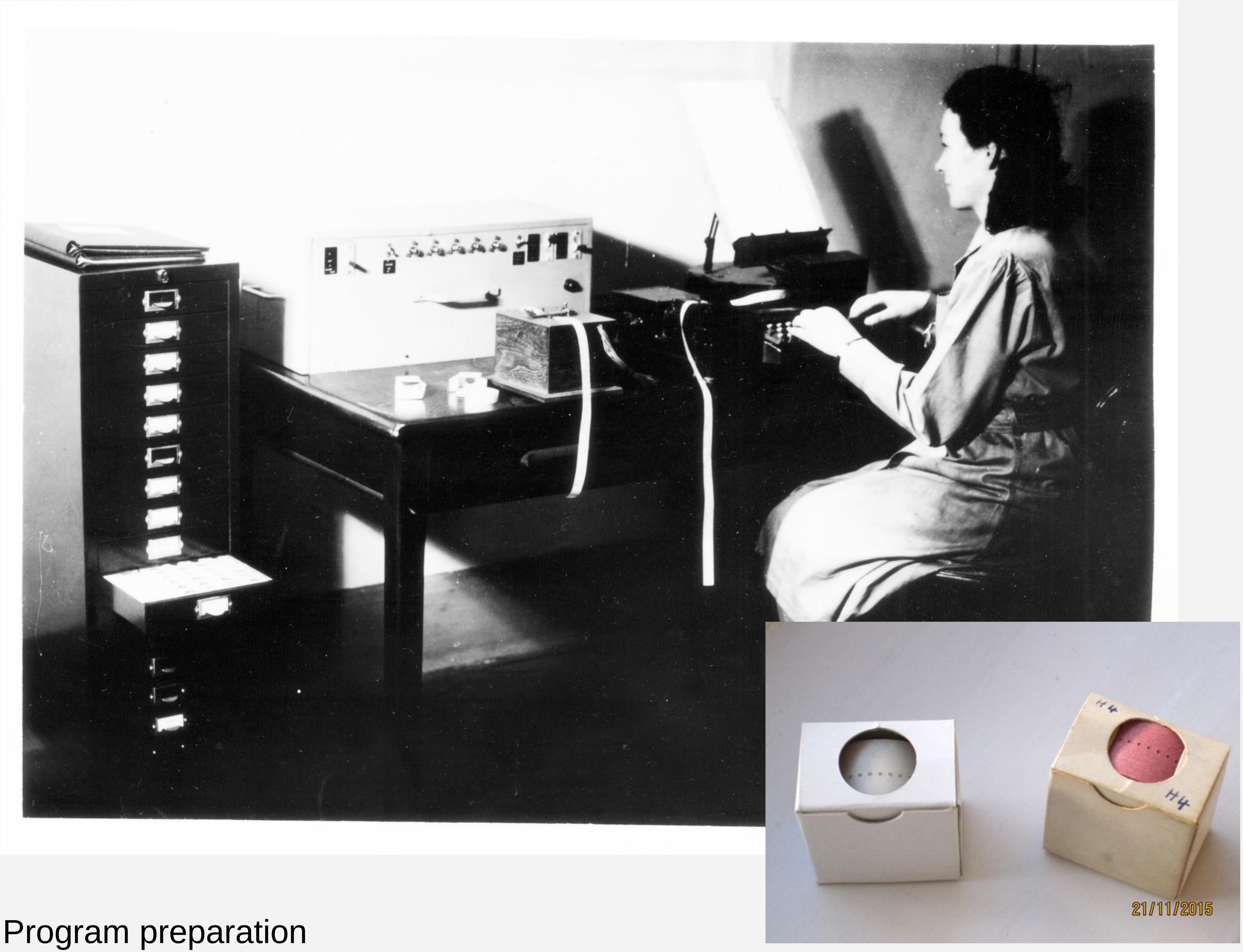


```
      92   G 113 S   branch if |η'-η|-e < 0
       3   T     S   ]
       4   A  42 L   ] η' to η
       5   T  40 L   ]
       6   E  73 S   repeat iteration
  152 →7   T     S   ]
  153 →8   A  33 S   ] x1 to x0
       9   T  34 S   ]
     100   A  35 S   ] x2 to x1          all scaled by 2^-11.10
       1   U  33 S   ]
       2   A  52 S   x2+δx to x2 ?
     3 4   A  38 L   ] y1 to y0
     4 5   T  36 L   ]
     5 6   A  42 L   ] y2 to y1
     6 7   T  38 L   ]
     7 8   H 119 S   ]
     8 9   V  33 S   ] 10x1.2^-11.10 to 44S
     9 110 L   4 S   ]
   110 1   T  44 S   ]                    3
     1 2   E  74 S   repeat
     2 3   T     S   ] branch to print y0
     3 4   E 127 S   ]
```

Wilkes' Airy Program June-July 1949

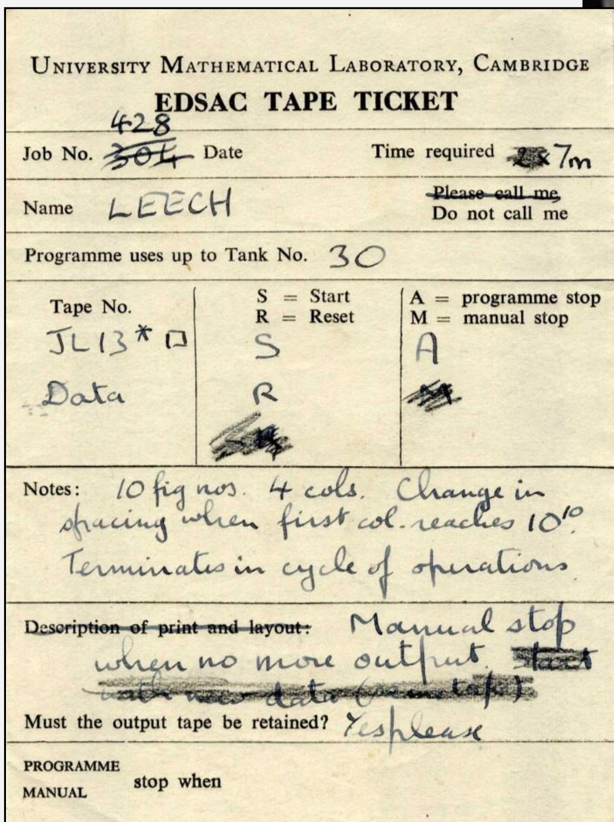


David Wheeler: Initial Orders 1 → Initial Orders 2



Program preparation

A Computing Service, January 1950



UNIVERSITY MATHEMATICAL LABORATORY, CAMBRIDGE

EDSAC TAPE TICKET

Job No. ~~304~~ 428 Date Time required ~~2~~ 7m

Name LEECH ~~Please call me~~ Do not call me

Programme uses up to Tank No. 30

Tape No.	S = Start R = Reset	A = programme stop M = manual stop
JL13* □	S	A
Data	R	

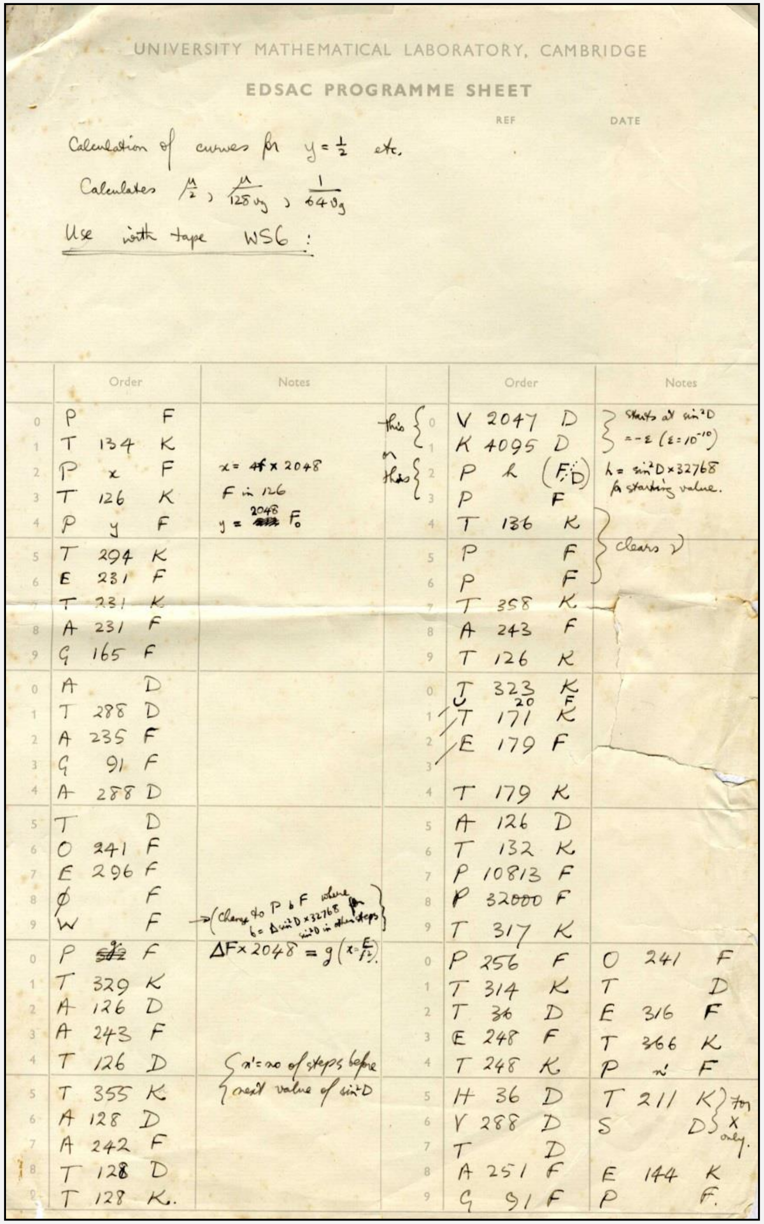
Notes: 10 fig nos. 4 cols. Change in spacing when first col. reaches 10^{10}. Terminates in cycle of operations.

~~Description of print and layout:~~ Manual stop when no more output.

Must the output tape be retained? Yes please

PROGRAMME
MANUAL stop when

The job ticket and queue



UNIVERSITY MATHEMATICAL LABORATORY, CAMBRIDGE

EDSAC PROGRAMME SHEET

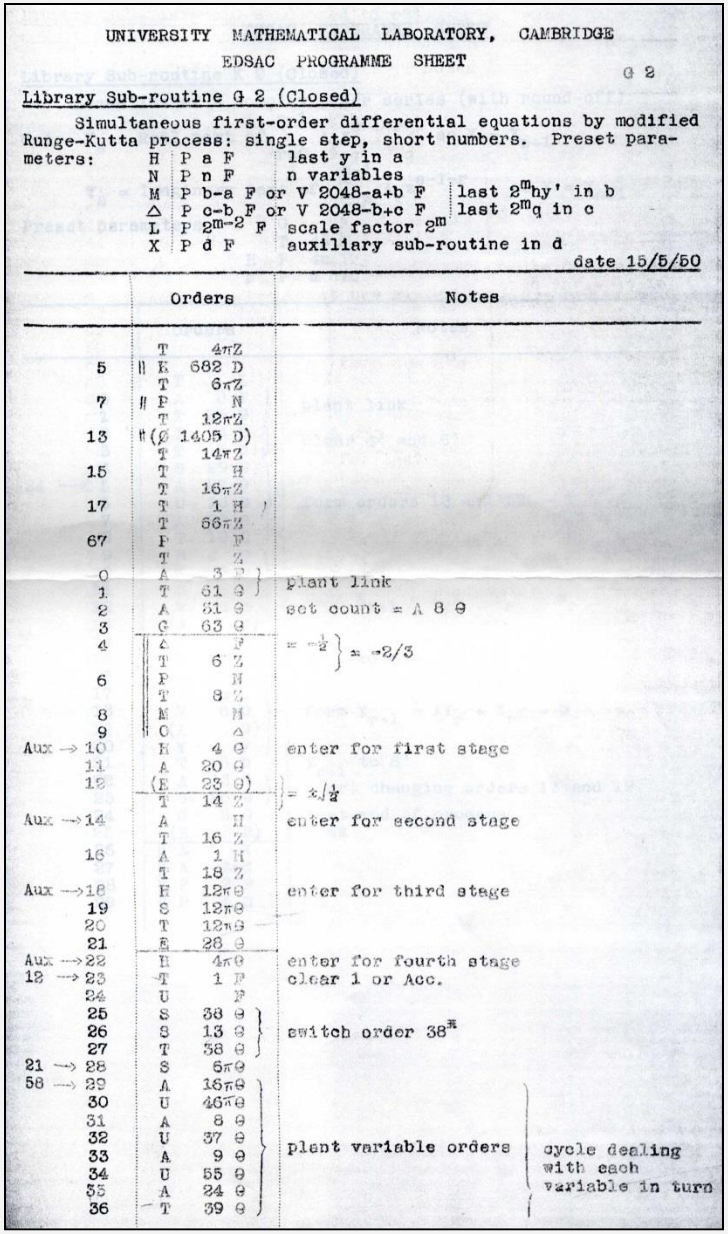
REF DATE

Calculation of curves for $y = \frac{1}{2}$ etc.

Calculates $\mu/2$, $\frac{\mu}{128 v_g}$, $\frac{1}{64 v_g}$

Use with tape WS6 :

	Order	Notes			Order	Notes
0	P F		this	0	V 2047 D	starts at $\sin^2 D$
1	T 134 K			1	K 4095 D	$= -\varepsilon$ ($\varepsilon = 10^{-10}$)
2	P x F	$x = 4f \times 2048$	or this	2	P h (F; D)	$h = \sin^2 D \times 32768$
3	T 126 K	F in 126		3	P F	for starting value.
4	P y F	$y = 2048\,F_0$		4	T 136 K	
5	T 294 K			5	P F	clears 2
6	E 231 F			6	P F	
7	T 231 K			7	T 358 K	
8	A 231 F			8	A 243 F	
9	G 165 F			9	T 126 K	
0	A D			0	J 323 K (20 F)	
1	T 288 D			1	T 171 K	
2	A 235 F			2	E 179 F	
3	G 91 F			3		
4	A 288 D			4	T 179 K	
5	T D			5	A 126 D	
6	O 241 F			6	T 132 K	
7	E 296 F			7	P 10813 F	
8	Ø F			8	P 32000 F	
9	W F	(Change to P b F where $b = \Delta\sin^2 D \times 32768$ for $\sin^2 D$ in other steps)		9	T 317 K	
0	P ~~512~~ g F	$\Delta F \times 2048 = g\left(x = \frac{F}{12}\right)$		0	P 256 F	O 241 F
1	T 329 K			1	T 314 K	T D
2	A 126 D			2	T 36 D	E 316 F
3	A 243 F			3	E 248 F	T 366 K
4	T 126 D	x' = no of steps before		4	T 248 K	P x' F
5	T 355 K	next value of $\sin^2 D$		5	H 36 D	T 211 K for
6	A 128 D			6	V 288 D	S D X only.
7	A 242 F			7	T D	
8	T 128 D			8	A 251 F	E 144 K
9	T 128 K.			9	G 91 F	P F.



UNIVERSITY MATHEMATICAL LABORATORY, CAMBRIDGE

EDSAC PROGRAMME SHEET

G 2

Library Sub-routine G 2 (Closed)

Simultaneous first-order differential equations by modified Runge-Kutta process: single step, short numbers. Preset parameters:

H	P a F	last y in a	
N	P n F	n variables	
M	P b-a F or V 2048-a+b F		last $2^m hy'$ in b
$\triangle$	P c-b F or V 2048-b+c F		last $2^m q$ in c
L	P 2^{m-2} F	scale factor 2^m	
X	P d F	auxiliary sub-routine in d	

date 15/5/50

	Orders	Notes
	T 4πZ	
5	‖ E 682 D	
	T 6πZ	
7	‖ P N	
	T 12πZ	
13	‖ (Ø 1405 D)	
	T 14πZ	
15	T H	
	T 16πZ	
17	T 1 H	
	T 56πZ	
67	P F	
	T Z	
0	A 3 F	plant link
1	T 61 θ	
2	A 51 θ	set count = A 8 θ
3	G 63 θ	
4	$\triangle$ F	$= -\frac{1}{2}$ } $= -2/3$
	T 6 Z	
6	P M	
	T 8 Z	
8	M M	
9	C $\triangle$	
Aux → 10	H 4 θ	enter for first stage
11	A 20 θ	
12	(E 23 θ)	$= \pm\sqrt{\frac{1}{2}}$
	T 14 Z	
Aux → 14	A H	enter for second stage
	T 16 Z	
16	A 1 H	
	T 18 Z	
Aux → 18	H 12πθ	enter for third stage
19	S 12πθ	
20	T 12πθ	
21	E 28 θ	
Aux → 22	H 4πθ	enter for fourth stage
12 → 23	T 1 F	clear 1 or Acc.
24	U F	
25	S 38 θ	switch order 38*
26	S 13 θ	
27	T 38 θ	
21 → 28	S 6πθ	
58 → 29	A 16πθ	plant variable orders — cycle dealing with each variable in turn
30	U 46πθ	
31	A 8 θ	
32	U 37 θ	
33	A 9 θ	
34	U 55 θ	
35	A 24 θ	
36	T 39 θ	

Program coding forms and MS conventions

Debugging by peeping

Sakharov, A. D. 1948 *J. Exp. Theor. Phys.* **18**, 631.
Schafroth, R. 1949 *Helv. Phys. Acta*, **22**, (iv), 392.
Schwinger, J. S. 1949 *Phys. Rev.* **76**, 790.
Streib, J. F., Fowler, W. A. & Lauritsen, C. C. 1941 *Phys. Rev.* **59**, 523.
Stuckelberg, E. C. G. 1938 *Helv. Phys. Acta*, **11**, 225.
Thomas, R. 1940 *Phys. Rev.* **58**, 714.
Tomlinson, E. P. 1941 *Phys. Rev.* **60**, 159 (A).
Ward, J. C. 1950 *Phys. Rev.* **78**, 182.

The diagnosis of mistakes in programmes on the EDSAC

By S. Gill
Mathematical Laboratory, University of Cambridge

(*Communicated by D. R. Hartree, F.R.S.—Received* 13 *December* 1950)

This paper describes methods developed at the Cambridge University Mathematical Laboratory for the speedy diagnosis of mistakes in programmes for an automatic high-speed digital computer. The aim of these methods is to avoid undue wastage of machine time, and a principal feature is the provision of several standard routines which may be used in conjunction with faulty programmes to check the operation of the latter. Two of these routines are considered in detail, and the others are briefly described.

1. Introduction

Two kinds of mistakes, or blunders, arise in the use of an automatic digital computing machine: (i) those resulting from faults in the machine itself, and (ii) those arising because the orders or data presented to the machine are not those required to obtain the results sought. This paper is entirely concerned with mistakes of the second kind, and describes methods employed for dealing with such mistakes on the EDSAC at the Cambridge University Mathematical Laboratory. Although it is written with special reference to this machine, much of the subject-matter is in principle more generally applicable.

Programmes are presented to the EDSAC in the form of punched tape, the entries on which are converted into orders and numbers by the machine as the tape is read. This process has been described in a paper by Wheeler (1950), and it will be assumed that the reader is already acquainted with that paper and the various technical terms employed therein. The order code of the machine is repeated here in appendix 1 for convenience.

It is natural at first to dismiss mistakes in programming as an inevitable but temporary evil, due to lack of experience, and to assume that if reasonable care is taken to prevent such mistakes occurring no other remedy is necessary. However, experience with the EDSAC has shown that although a high proportion of mistakes can be removed by preliminary checking, there frequently remain mistakes which could only have been detected in the early stages by prolonged and laborious study.

Appendix 3

Library sub-routine C 11

location with respect to first order	order		notes	
	G	K	control combination	
0	$(P$	$F)$	storage for sign of $C(A)$	(note 2)
1	$(P$	$F)$	storage for $C(A)$	
2	θ	F		constants
3	Δ	F	$=-1/2$	
4	A	F	$=-1/4$	
5	Q	F	$=+1/16$	
18 → 6	A	$4\,\theta$	form new select order (note 4) IX	
7	O	$2\,\theta$	teleprinter carriage return	X
8	O	$3\,\theta$	teleprinter line feed	
31 → 9	U	$11\,\theta$	place select order	VIII
10	S	$11\,\theta$		IX
11	$(Z$	$F)$	select order	I
12	U	$22\,\theta$	place current order	
Enter → 13	O	$22\,\theta$	print function letter (note 6) II	
14	S	θ	$3/16 \leqslant x < 4/16$	(note 4) III
15	A	$4\,\theta$	$-1/16 \leqslant x < 0$	
16	E	$19\,\theta$		
17	A	$5\,\theta$	$0 \leqslant x < 1/16$	
18	E	$6\,\theta$		
16 → 19	U	θ	clear top 17 digits of accumulator	IV
20	S	θ		
21	A	$1\,\theta$	restore $C(A)$	
22	$(K$	$3000\,F)$	current order (note 6) V	
23	U	$1\,\theta$	store $C(A)$ VI	
24	E	$26\,\theta$	test sign of $C(A)$	VII
25	A	$3\,\theta$	add $-1/2$ if $C(A)<0$	
24 → 26	S	$1\,\theta$	subtract $C(A)$	
27	U	θ	store sign of $C(A)$ (note 2)	
28	S	θ		
29	A	$11\,\theta$	select order to accumulator	VIII
30	A	$2\,F$	add unity to address	
31	G	$9\,\theta$		
	E	$13\,Z$		

Followed on tape by

$E\ m\ F$ punched by user

When this has been read, control is switched to order 13 of this routine, with $E\,m\,F$ in the accumulator.

Notes

(1) The notation described in appendix 2, note 1, is used.

(2) As in $C\,1$, all working space must lie within the routine itself. This includes a location for the storage of the 17 digits, referred to as $C(A)$, which would be at the top (most significant end) of the accumulator if the original programme were operating directly, and also a location for recording separately the sign of $C(A)$. The latter is coded thus: 0 if $C(A) \geqslant 0$; $-1/2$ if $C(A) < 0$.

(3) The whole of the accumulator, except the top 17 digits, must remain undisturbed from the obeying of one current order to that of the next. Hence

Debugging: Postmortem and Checking Routines (Stanley Gill)

Sakharov, A. D. 1948 *J. Exp. Theor. Phys.* **18**, 631.
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Streib, J. F., Fowler, W. A. & Lauritsen, C. C. 1941 *Phys. Rev.* **59**, 523.
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Thomas, R. 1940 *Phys. Rev.* **58**, 714.
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The diagnosis of mistakes in programmes on the EDSAC

By S. Gill
Mathematical Laboratory, University of Cambridge

(*Communicated by D. R. Hartree, F.R.S.—Received* 13 *December* 1950)

This paper describes methods developed at the Cambridge University Mathematical Laboratory for the speedy diagnosis of mistakes in programmes for an automatic high-speed digital computer. The aim of these methods is to avoid undue wastage of machine time, and a principal feature is the provision of several standard routines which may be used in conjunction with faulty programmes to check the operation of the latter. Two of these routines are considered in detail, and the others are briefly described.

1. Introduction

Two kinds of mistakes, or blunders, arise in the use of an automatic digital computing machine: (i) those resulting from faults in the machine itself, and (ii) those arising because the orders or data presented to the machine are not those required to obtain the results sought. This paper is entirely concerned with mistakes of the second kind, and describes methods employed for dealing with such mistakes on the EDSAC at the Cambridge University Mathematical Laboratory. Although it is written with special reference to this machine, much of the subject-matter is in principle more generally applicable.

Programmes are presented to the EDSAC in the form of punched tape, the entries on which are converted into orders and numbers by the machine as the tape is read. This process has been described in a paper by Wheeler (1950), and it will be assumed that the reader is already acquainted with that paper and the various technical terms employed therein. The order code of the machine is repeated here in appendix 1 for convenience.

It is natural at first to dismiss mistakes in programming as an inevitable but temporary evil, due to lack of experience, and to assume that if reasonable care is taken to prevent such mistakes occurring no other remedy is necessary. However, experience with the EDSAC has shown that although a high proportion of mistakes can be removed by preliminary checking, there frequently remain mistakes which could only have been detected in the early stages by prolonged and laborious study.

In order to carry out this example, $C\,11$ would first have to be placed in the store and then directed to examine the initial orders, starting at a suitable point, say order 34. If $C\,11$ is placed in locations from 100 onwards, the complete tape would consist of

$$T\ 100\ K$$
sub-routine $C\,11$
$$E\ \ 34\ F$$

followed by the symbols to be read during the test.

Below is shown the result. Only the letters on the left are actually produced by the machine, the other columns being given for the guidance of the reader, but it will be seen that the letters themselves are in fact sufficient to show the course of the programme. The time required, including tape input, to obtain the results shown is less than a minute.

symbols printed by teleprinter	positions of corresponding orders	symbols read from tape by *I* order
IARTE	34 to 38	*T*
TIASG	8 to 12	6
ARVLTIASG	4 to 12	0
ARVLTIASGLSESATAE	4 to 20	*K*
AE	30 to 31	
TE	25 to 26	
IARTE	34 to 38	*G*
TIASGLSESATAE	8 to 20	*K*
AEATIARTE	30 to 38	*A*
TIASG	8 to 12	8
ARVLTIASGLSESATAE	4 to 20	π
AE	27 to 28	
TIASGLSE	8 to 15	θ
ATAAATAATE	17 to 26	
IARTE	34 to 38	*T*
TIASG	8 to 12	3
ARVLTIASG	4 to 12	1
ARVLTIASG	4 to 12	0
ARVLTIASGLSE	4 to 15	*D*
ATAAATAATE	17 to 26	

etc.

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Wilkes, M. V. 1949 *J. Sci. Instrum.* **26**, 217.
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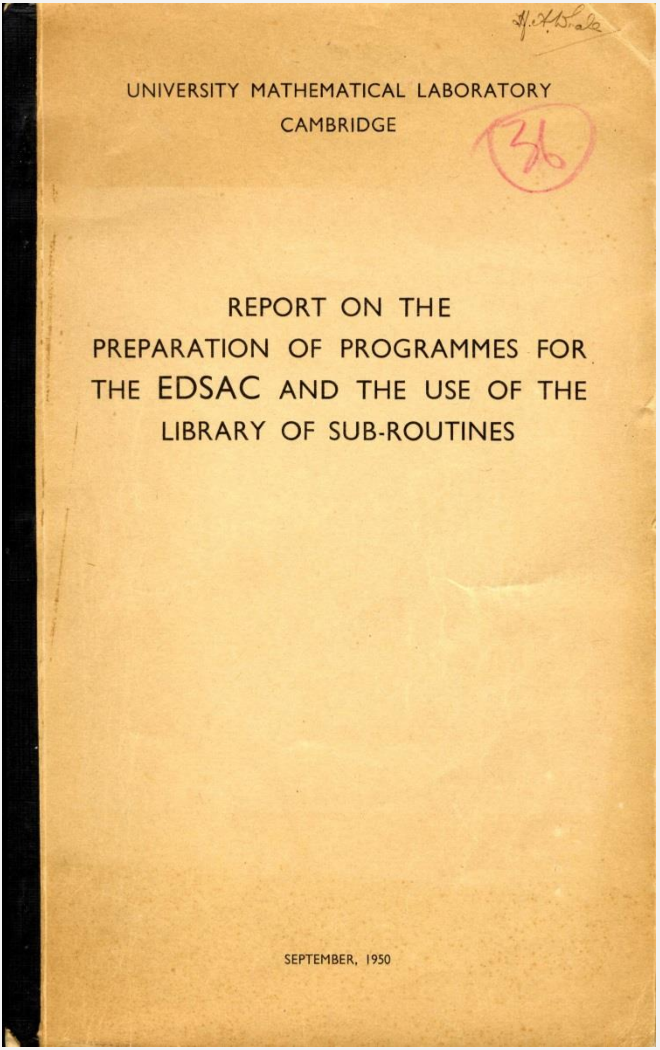
Wilkes, M. V., Wheeler, D. J. & Gill, S. 1951 *The preparation of programs for an electronic digital computer, with special reference to the EDSAC and the use of a library of subroutines.* Cambridge, Mass: Addison-Wesley Press Inc.

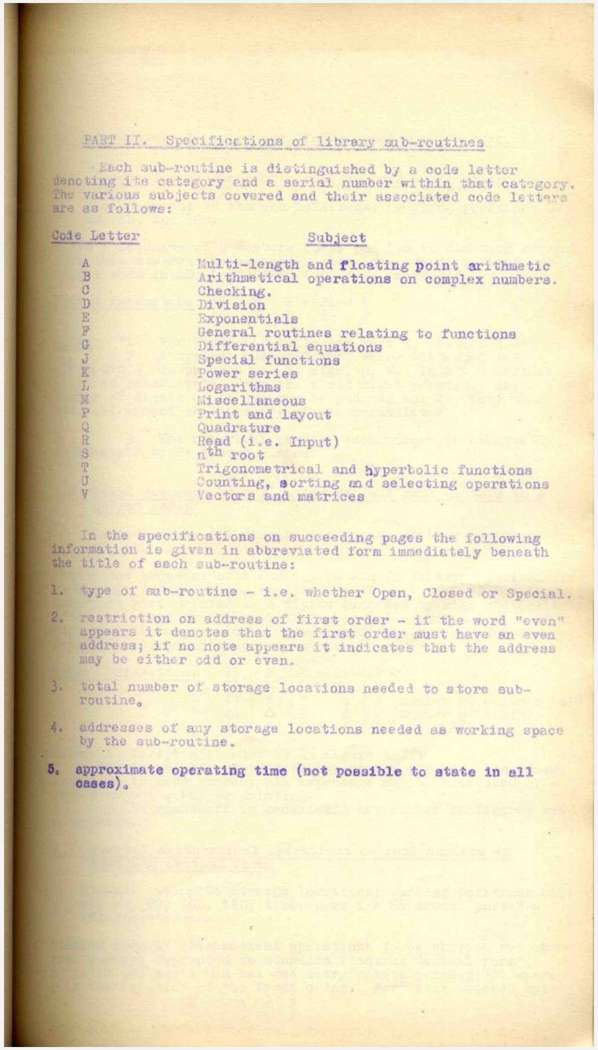
Debugging: Postmortem and Checking Routines (Stanley Gill)



UNIVERSITY MATHEMATICAL LABORATORY
CAMBRIDGE

REPORT ON THE PREPARATION OF PROGRAMMES FOR THE EDSAC AND THE USE OF THE LIBRARY OF SUB-ROUTINES

SEPTEMBER, 1950



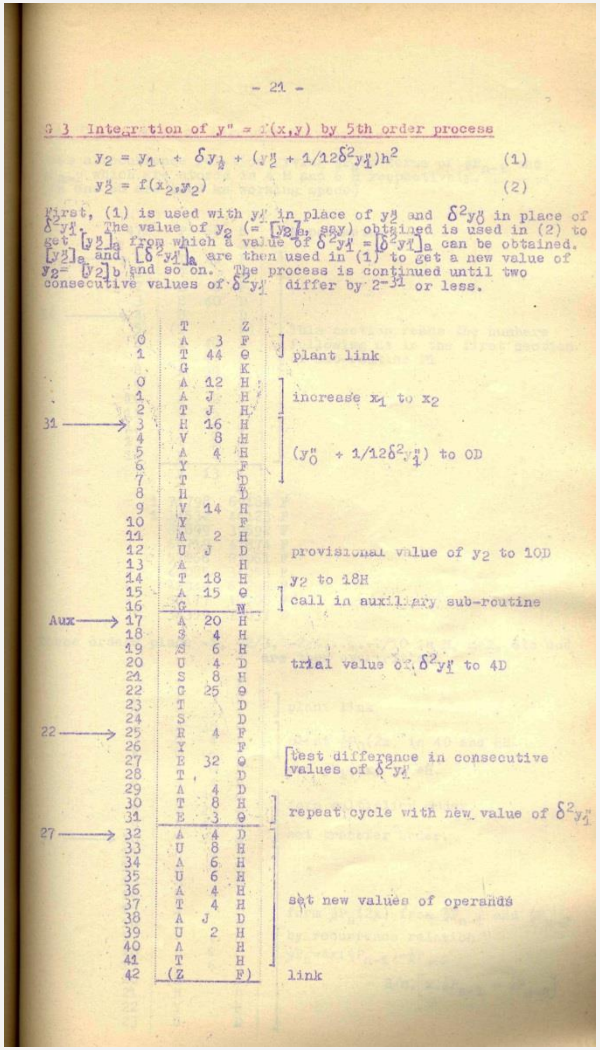
PART II. Specifications of library sub-routines

Each sub-routine is distinguished by a code letter denoting its category and a serial number within that category. The various subjects covered and their associated code letters are as follows:

Code Letter	Subject
A	Multi-length and floating point arithmetic
B	Arithmetical operations on complex numbers.
C	Checking.
D	Division
E	Exponentials
F	General routines relating to functions
G	Differential equations
J	Special functions
K	Power series
L	Logarithms
M	Miscellaneous
P	Print and layout
Q	Quadrature
R	Read (i.e. Input)
S	n^{th} root
T	Trigonometrical and hyperbolic functions
U	Counting, sorting and selecting operations
V	Vectors and matrices

In the specifications on succeeding pages the following information is given in abbreviated form immediately beneath the title of each sub-routine:

1. type of sub-routine - i.e. whether Open, Closed or Special.
2. restriction on address of first order - if the word "even" appears it denotes that the first order must have an even address; if no note appears it indicates that the address may be either odd or even.
3. total number of storage locations needed to store sub-routine.
4. addresses of any storage locations needed as working space by the sub-routine.
5. **approximate operating time (not possible to state in all cases).**



- 21 -

G 3 Integration of y" = f(x,y) by 5th order process

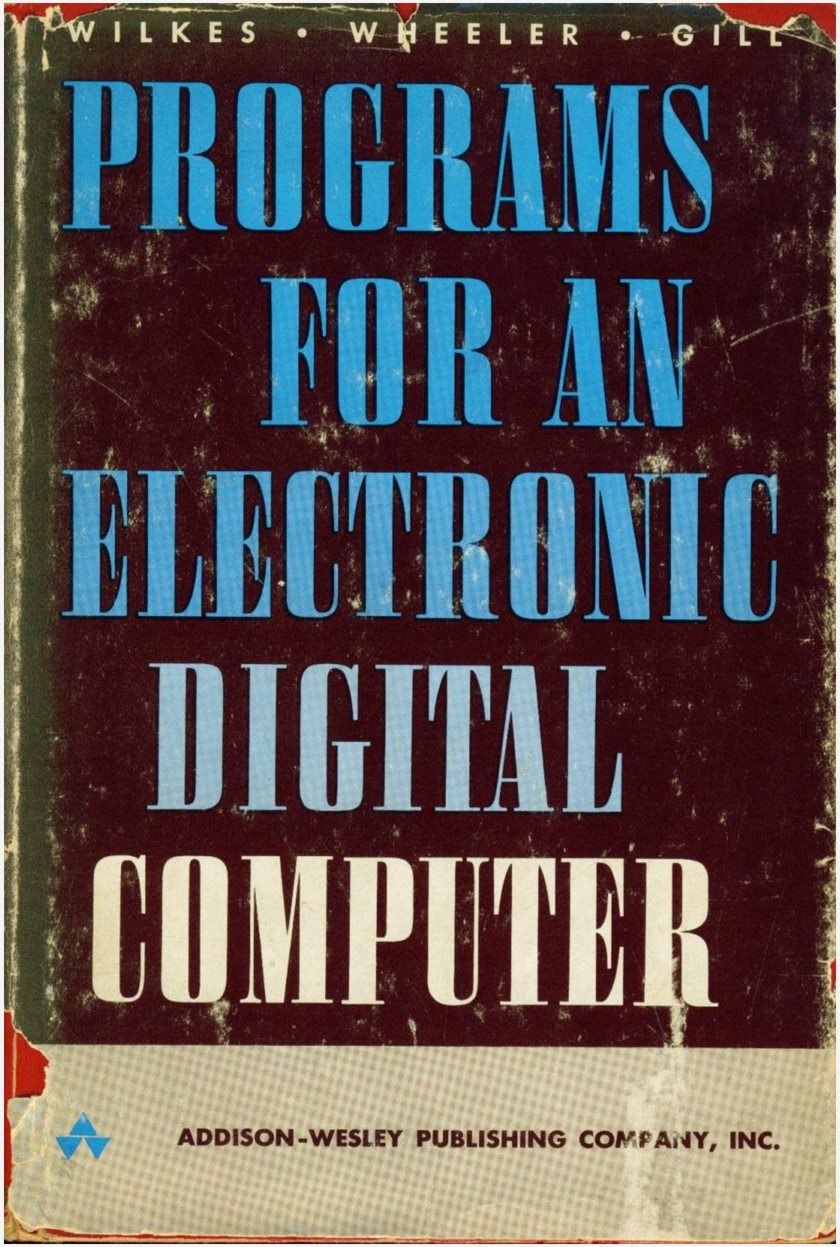
$$y_2 = y_1 + \delta y_{\frac{1}{2}} + (y''_{\frac{1}{2}} + 1/12\,\delta^2 y''_1)h^2 \qquad (1)$$

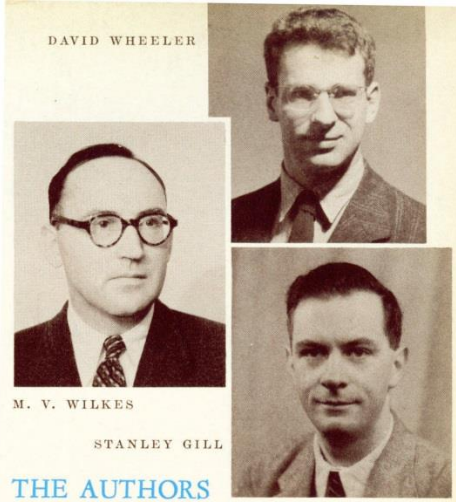
$$y''_2 = f(x_2, y_2) \qquad (2)$$

First, (1) is used with y''_1 in place of $y''_{\frac{1}{2}}$ and $\delta^2 y''_0$ in place of $\delta^2 y''_1$. The value of y_2 (= $[y_2]_a$, say) obtained is used in (2) to get $[y''_2]_a$ from which a value of $\delta^2 y''_1 = [\delta^2 y''_1]_a$ can be obtained. $[y''_2]_a$ and $[\delta^2 y''_1]_a$ are then used in (1) to get a new value of $y_2 = [y_2]_b$ and so on. The process is continued until two consecutive values of $\delta^2 y''_1$ differ by 2^{-31} or less.

```
             T      Z
        0    A   3  F    ] plant link
        1    T  44  θ    ]
             G      K
        0    A  12  H    ] increase x1 to x2
        1    A   J  H    ]
        2    T   J  H    ]
31 -->  3    H  16  H    ]
        4    V   8  H    ] (y"0 + 1/12δ²y"1) to 0D
        5    A   4  H    ]
        6    Y      F    ]
        7    T      D    ]
        8    H      D
        9    V  14  H
       10    Y      F
       11    A   2  H
       12    U   J  D      provisional value of y2 to 10D
       13    A      H
       14    T  18  H      y2 to 18H
       15    A  15  θ    ] call in auxiliary sub-routine
       16    G      N    ]
Aux--> 17    A  20  H
       18    S   4  H
       19    S   6  H
       20    U   4  D      trial value of δ²y"1 to 4D
       21    S   8  H
       22    G  25  θ
       23    T      D
       24    S      D
22 --> 25    R   4  F
       26    Y      F      [test difference in consecutive
       27    E  32  θ      [values of δ²y"1
       28    T      D
       29    A   4  D
       30    T   8  H    ]
       31    E   3  θ    ] repeat cycle with new value of δ²y"1
27 --> 32    A   4  D    ]
       33    U   8  H    ]
       34    A   6  H    ]
       35    U   6  H    ]
       36    A   4  H    ]
       37    T   4  H    ] set new values of operands
       38    A   J  D    ]
       39    U   2  H    ]
       40    A      H    ]
       41    T      H    ]
       42   (Z      F)     link
```

Report on Programming, September 1950





THE AUTHORS

M. V. Wilkes took the Mathematical Tripos at Cambridge University in 1934, and later did graduate work on radio wave propagation at the Cavendish Laboratory. During the war he was engaged in radar and operational research, and when the war was over was appointed Director of the University Mathematical Laboratory. He has visited the United States on a number of occasions and is well known there in the computer field.

David Wheeler entered the field of automatic computing in 1948. During the years 1948–51 he did graduate work in programming and numerical analysis in the Mathematical Laboratory at Cambridge. He obtained his Ph.D. degree in 1951 and a research fellowship at Trinity College later in the same year.

During the years 1951–53 he was Visiting Assistant Professor at the University of Illinois. He returned to England in 1953 and now holds a staff appointment in the Mathematical Laboratory.

Stanley Gill entered the electronic computer field in 1947 while at the National Physical Laboratory, where he joined the design team planning the Pilot Model A.C.E. Later he became a programmer at Cambridge, where he obtained his Ph.D. degree in 1953 for research into methods of applying the EDSAC to problems in mathematics and physics. This work included the introduction of mistake diagnostic routines and the development of a particular form of the Runge-Kutta process for the solution of differential equations.

Dr. Gill spent 18 months in the United States during 1953–54, where he succeeded Dr. Wheeler in the position of Visiting Assistant Professor at the University of Illinois; he also lectured at Summer Session courses at the Massachusetts Institute of Technology. He is now head of the Computing Research Group of Ferranti Ltd., in London.

PRINTED IN U.S.A.

Wilkes, Wheeler & Gill, 1951; 2nd ed. 1957

Applications

The Priorities Committee: J.C.P. Miller, Stanley Gill, Beatrice Worsley

AM/4/56/391

SUMMER SCHOOL IN PROGRAMME DESIGN FOR
AUTOMATIC DIGITAL COMPUTING MACHINES

17 - 28 September 1956

T I M E - T A B L E

The main course of lectures P.1 to P.13 is devoted to programming. Other lectures cover allied topics including numerical analysis.

Monday, 17 September

2.30	Registration	
4.30	General Introduction	D.R. Hartree

Tuesday, 18 September

9.30	P.1 §1 - §9	D.R. Hartree
11.20	Practical Classes in Programming 1-A8, 38	
2.30	P.2 - §13	D.R. Hartree

Wednesday, 19 September

9.30	Practical Classes in Programming - C, D	
11.20	P.3 - §21	D.R. Hartree
2.30	Practical Classes in Programming 2	

Thursday, 20 September

9.30	P.4 - §28	D.R. Hartree
11.20	Procedure in preparing and running a programme on the EDSAC	E.N. Mutch
2.30	Practical Classes in Programming 3	

Friday, 21 September

9.30	P.5	D.R. Hartree
11.20	Practical Classes in Programming	
2.30	P.6	D.R. Hartree
* 4.30	Logical Design of the EDSAC	E.N. Mutch
* 5.15	Logical Design of the EDSAC 2	W. Renwick

Saturday, 22 September

9.30	P.7	D.R. Hartree
11.20	Practical Classes in Programming	

Monday, 24 September

9.30	P.8	E.N. Mutch
11.20	Practical Classes in Programming	
2.30	Numerical analysis (1)	J.C.P. Miller
* 4.30	Organisation of a Computing Centre	M.V. Wilkes

Tuesday, 25 September

9.30	P.9	M.V. Wilkes
11.20	Practical Classes in Programming	
2.30	Numerical analysis (2)	J.C.P. Miller

Wednesday, 26 September

9.30	P.10	M.V. Wilkes
11.20	Practical Classes in Programming	
2.30	Numerical analysis (3)	J.C.P. Miller

* optional item

P.T.O.

AM/4/56/471

SUMMER SCHOOL IN PROGRAMME DESIGN
FOR AUTOMATIC DIGITAL COMPUTING MACHINES

17 - 28 September 1956

LIST OF STUDENTS

No.	Name	Place of Employment	Cambridge Address
64.	ABSALOM, Mr W.L.G.	A.E. Reed & Co., Ltd., Aylesford Paper Mills, Larkfield, Nr Maidstone, Kent.	6 Warkworth Street
42.	AGAR, Miss D.M.	English Electric Co. Ltd., Cambridge Road, Whetstone, Nr Leicester.	University Arms Hotel
67.	ALENIUS, Mr B.	Swedish Board for Computing Machinery, Stockholm, Sweden.	"Hillside", 13 Chesterton Lane
77.	BADCOCK, Miss P.M.J.H.	Ferranti Ltd., 21 Portland Place, London, W.1.	56 Jesus Lane
54.	BAINES, Dr A.	I.C.I. Ltd., General Chemicals Division, Research Department, Widnes.	Goldsborough Hotel
63.	BARCLAY, Mr W.	Middlesex County Council, Acton Technical College.	8 St Andrew's Hill
110.	BARNETT, Mrs L.	Cambridge University, M.R.C. Unit, Cavendish Laboratory.	58 Bateman Street
83.	BARRON, Mr D.W.		215 Queen Edith's Way.
101.	BEAVEN, Mr A.H.	Elliott Brothers (London) Ltd., Elstree Way, Borehamwood, Herts.	71 Jesus Lane
4.	BECHERET, Miss O.	E.D.F. 12 Place des Etats Unis, Paris, France.	56 Jesus Lane
98.	BECK, Mr F.	G.E.C. Ltd. Research Laboratories, North Wembley, Middx.	Garden House Ho
23.	BRAZIER, Mr D.E.	British Aero-Engines Ltd. Patchway, Nr Bristol.	2 Willis Road
5.	BROISE, Mr P.	E.D.F., 12 Place des Etats Unis, Paris, France.	27 Malcolm Stre
60.	BRYANT, Mr P.R.	G.E.C. Ltd. Research Laboratories, East Lane, Wembley.	6 North Terrace
107.	CARTZ, Mr L.	Cavendish Laboratory	60 Hertford Str
33.	CHAPPEY, Mr M.	Centre National d'Etudes des Telecommunications, 2 bis Avenue de la Republique, Issy les Moulineaux, (Seine) France.	"Hillside", 13 Chesterton Lane
34.	CHRIST, Miss G.M.	Cambridge University, Engineering Laboratory, Trumpington Street.	12 Mill Lane, 22 September 1 Huntingdon Road

P.T.O.

1956

John Leech

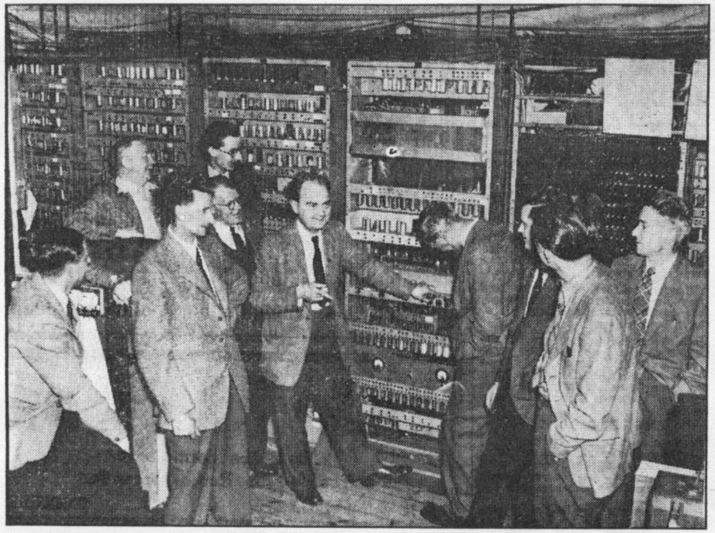
INTRODUCTION TO PROGRAMMING
FOR THE EDSAC

A supplement to
"The Preparation of Programs for
an Electronic Digital Computer"
by Wilkes, Wheeler and Gill
(Addison-Wesley Press, 1951)

UNIVERSITY MATHEMATICAL LABORATORY
CAMBRIDGE

Summer School in Programming, from September 1951

Machine Plays Noughts and Crosses



DR. J. C. P. Miller, Assistant Director of the University Mathematical Laboratory, talking to members of the Cambridge and District Amateur Radio Club at the Mathematical Laboratory, Corn Exchange Street, on Friday evening.

The visit was arranged to enable members to see the electronic calculating machine, popularly known as the "electronic brain," part of which is in the background in the top picture. The other picture shows the operating position of the machine into which problems are fed on specially prepared perforated tape on the left of the unit.

The machine was built in 1949, largely from Government surplus equipment due to lack of funds, and now serves all departments of the University. It is capable of doing 40,000 calculations per minute and uses 3,800 valves.

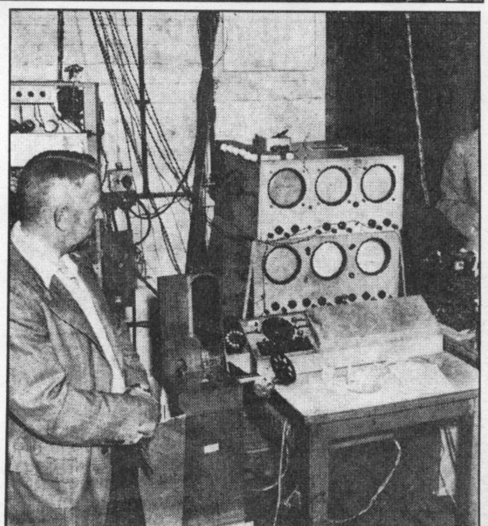
For the last three years a new machine of an even more complex nature, has been under construction and it will, it is hoped, be working in about another year.

Members of the Radio Club were fascinated to see Mr. E. N. Mutch, the operational manager, play noughts and crosses with the machine as well as do extremely complex calculations, which previously took many hours of tedious work.

FRED HOYLE WAS THERE.

When members arrived at the laboratory, Fred Hoyle, the famous cosmologist and broadcaster on astronomical subjects, was using the machine for calculations on the evolution of the stars.

A general description of the uses of the machine and how it works was given by Mr. Mutch, who was assisted in the actual demonstration by Mr. J. Leech, a research student. He stated that the name "electronic brain" was



Photos "Cambridge Daily News"

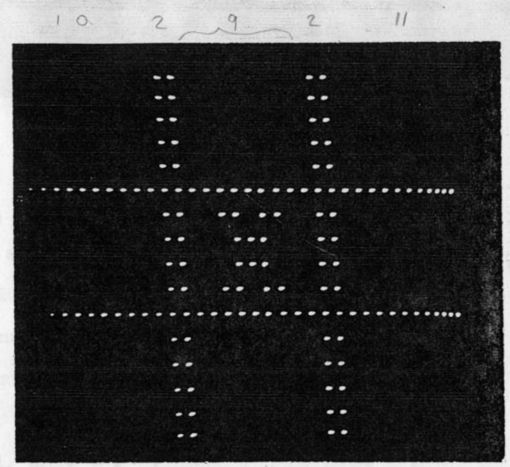
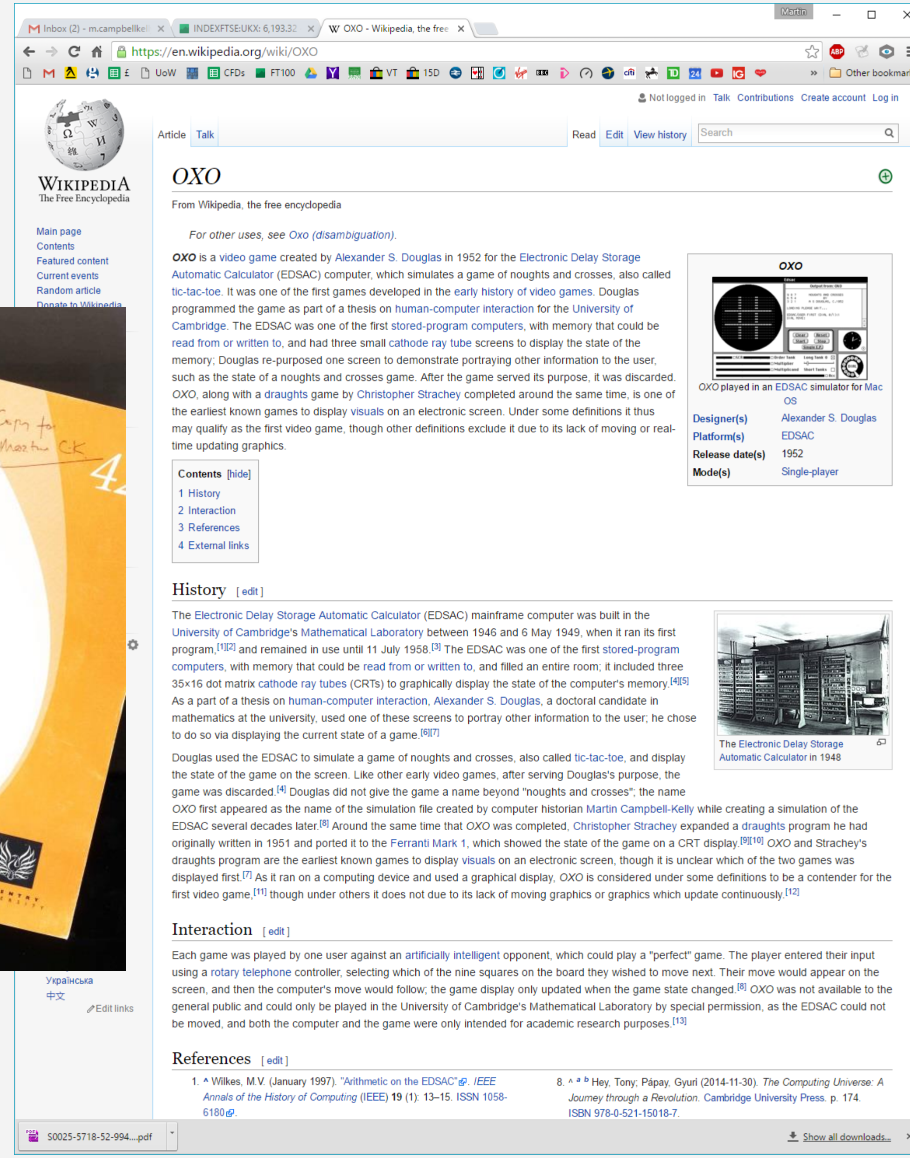


Diagram 1(c). The board with a cross in position 5.

9	8.	7.
6.	5.	4.
3.	2.	1.

Diagram 2(c). The numbering of the board.

Sandy Douglas' Noughts and Crosses



Sandy Douglas' Noughts and Crosses – History in the making

Prominent Users applying to the “Priorities Committee”

M. V. Wilkes: Atmospheric oscillations

R.A. Fisher (research student David Wheeler): Genetics

Peter Naur: Orbital calculations

S.F. Boys (research student A.S. Douglas): Wave mechanics

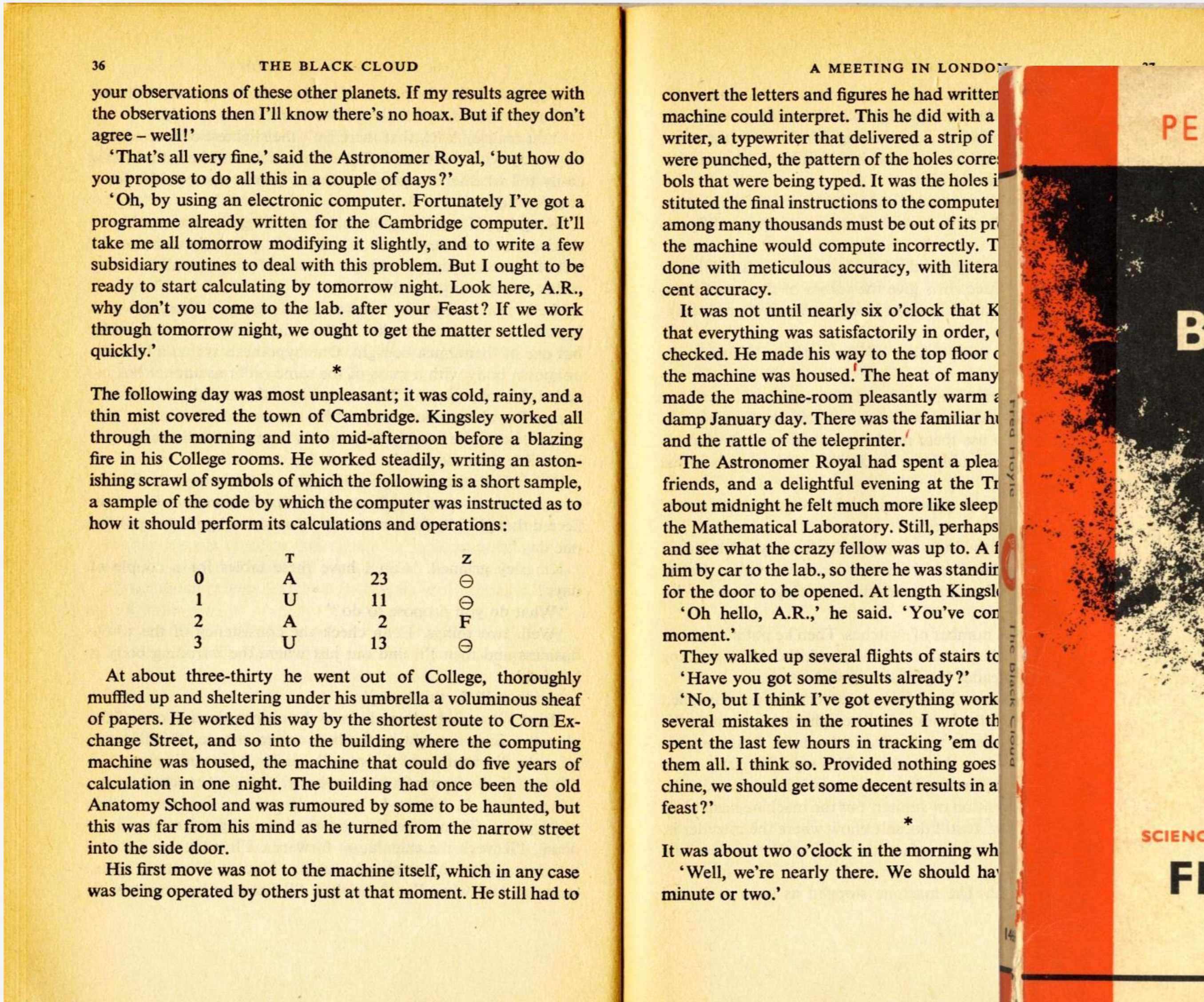
Fred Hoyle (research student Joyce Wheeler): Astrophysics

J.C. Kendrew (research student J.M. Bennett): Molecular biology (Nobel Laureate 1962)

Andrew Huxley: physiology (Nobel Laureate 1963)

Martin Ryle: Radio astronomy (Nobel Laureate 1974)

The EDSAC was a scientific instrument


36 THE BLACK CLOUD

your observations of these other planets. If my results agree with the observations then I'll know there's no hoax. But if they don't agree – well!'

'That's all very fine,' said the Astronomer Royal, 'but how do you propose to do all this in a couple of days?'

'Oh, by using an electronic computer. Fortunately I've got a programme already written for the Cambridge computer. It'll take me all tomorrow modifying it slightly, and to write a few subsidiary routines to deal with this problem. But I ought to be ready to start calculating by tomorrow night. Look here, A.R., why don't you come to the lab. after your Feast? If we work through tomorrow night, we ought to get the matter settled very quickly.'

*

The following day was most unpleasant; it was cold, rainy, and a thin mist covered the town of Cambridge. Kingsley worked all through the morning and into mid-afternoon before a blazing fire in his College rooms. He worked steadily, writing an astonishing scrawl of symbols of which the following is a short sample, a sample of the code by which the computer was instructed as to how it should perform its calculations and operations:

	T		Z
0	A	23	⊖
1	U	11	⊖
2	A	2	F
3	U	13	⊖

At about three-thirty he went out of College, thoroughly muffled up and sheltering under his umbrella a voluminous sheaf of papers. He worked his way by the shortest route to Corn Exchange Street, and so into the building where the computing machine was housed, the machine that could do five years of calculation in one night. The building had once been the old Anatomy School and was rumoured by some to be haunted, but this was far from his mind as he turned from the narrow street into the side door.

His first move was not to the machine itself, which in any case was being operated by others just at that moment. He still had to

A MEETING IN LONDON

convert the letters and figures he had written
machine could interpret. This he did with a
writer, a typewriter that delivered a strip of
were punched, the pattern of the holes corre
bols that were being typed. It was the holes i
stituted the final instructions to the compute
among many thousands must be out of its pr
the machine would compute incorrectly. T
done with meticulous accuracy, with litera
cent accuracy.

It was not until nearly six o'clock that K
that everything was satisfactorily in order,
checked. He made his way to the top floor
the machine was housed. The heat of many
made the machine-room pleasantly warm
damp January day. There was the familiar h
and the rattle of the teleprinter.

The Astronomer Royal had spent a plea
friends, and a delightful evening at the Tr
about midnight he felt much more like slee
the Mathematical Laboratory. Still, perhaps
and see what the crazy fellow was up to. A
him by car to the lab., so there he was standi
for the door to be opened. At length Kingsl

'Oh hello, A.R.,' he said. 'You've co
moment.'

They walked up several flights of stairs t

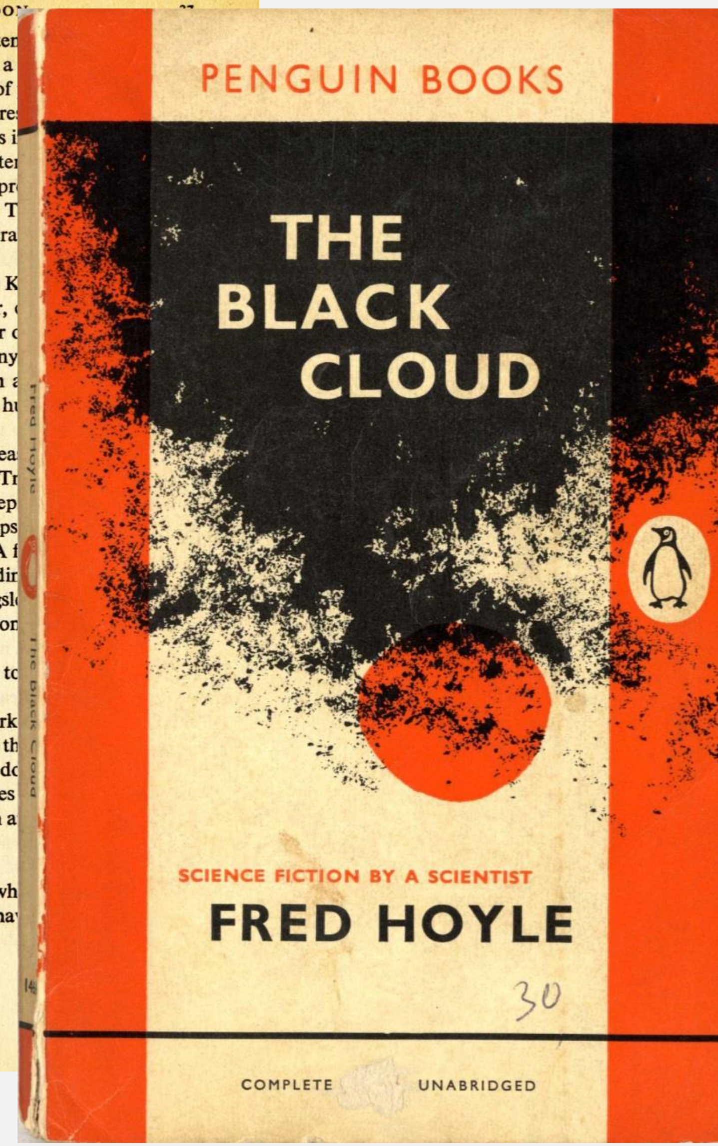
'Have you got some results already?'

'No, but I think I've got everything work
several mistakes in the routines I wrote th
spent the last few hours in tracking 'em d
them all. I think so. Provided nothing goes
chine, we should get some decent results in a
feast?'

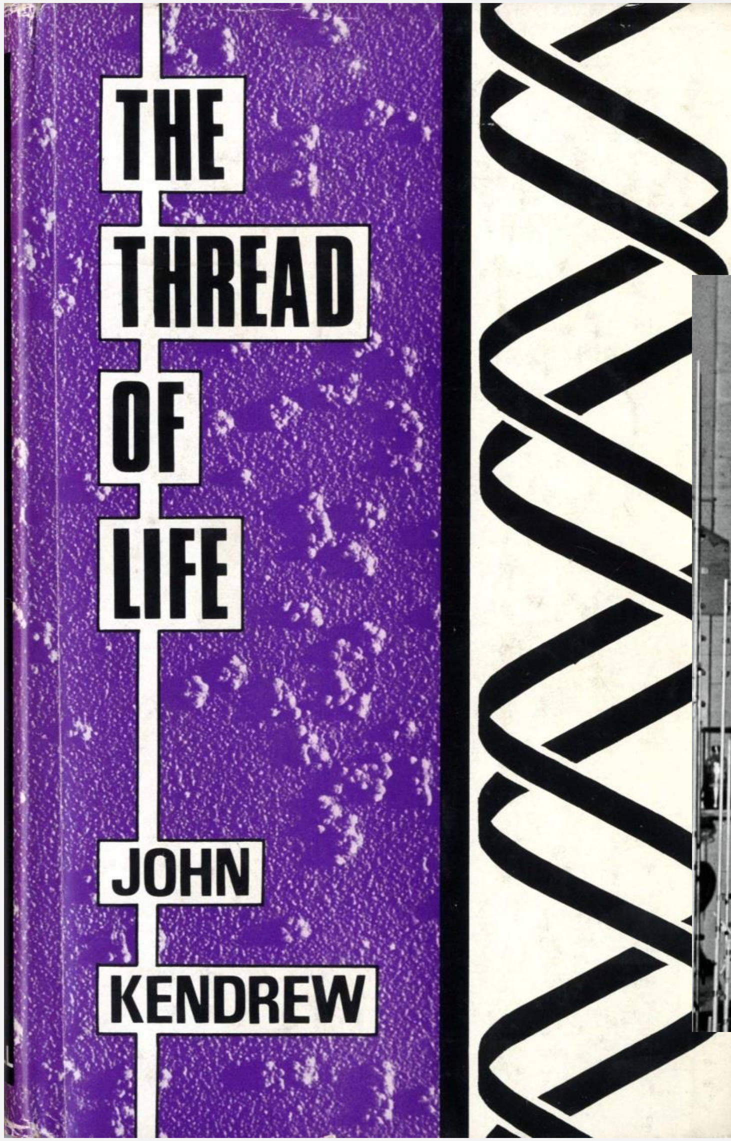
*

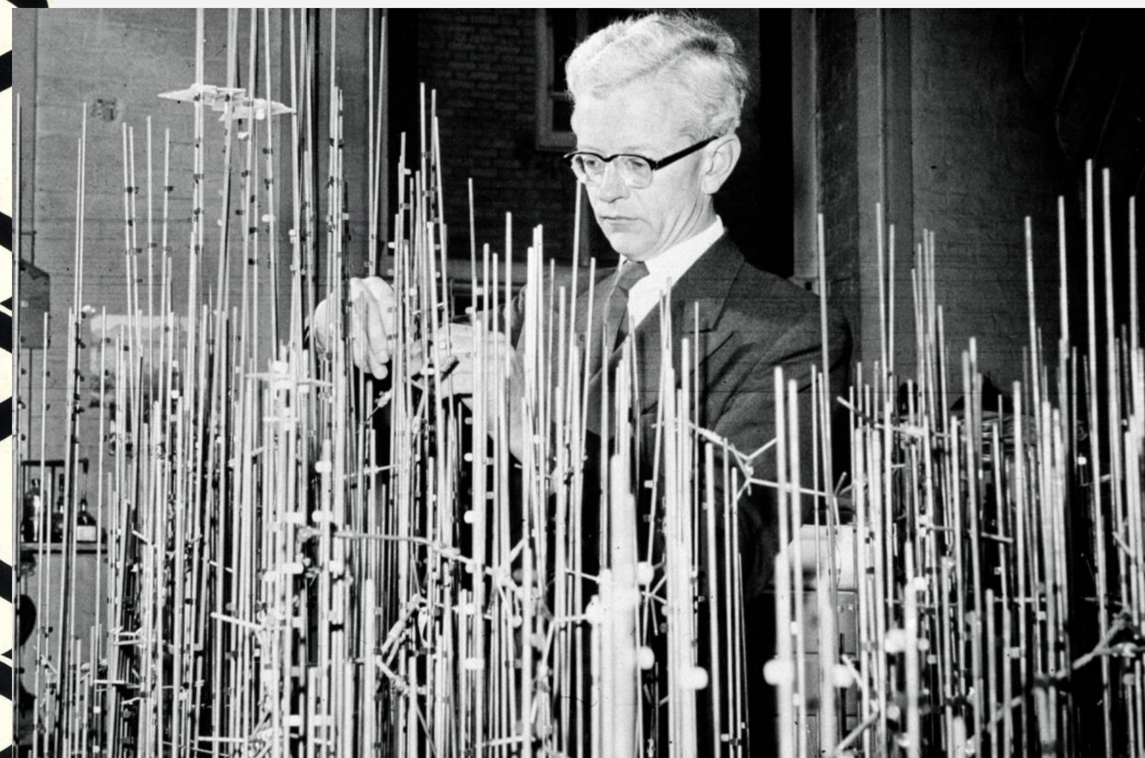
It was about two o'clock in the morning wh

'Well, we're nearly there. We should ha
minute or two.'



Fred Hoyle, The Black Cloud, 1957





Kendrew & Bennett: Crystallography and the road to DNA

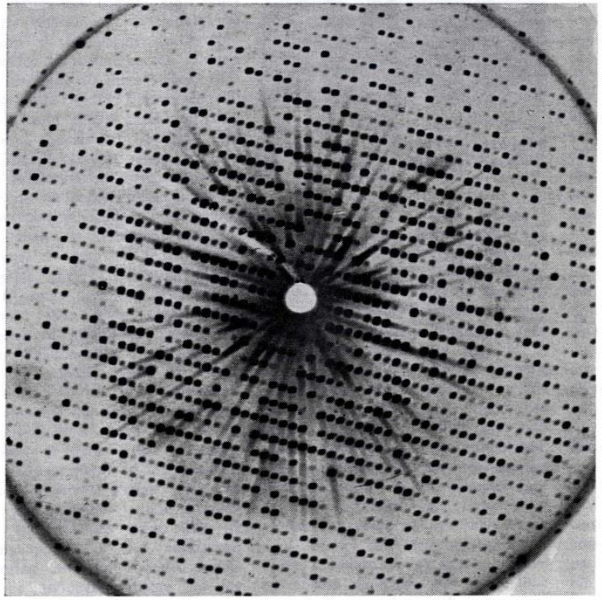


Fig. 1. X-ray precession photograph of a myoglobin crystal.

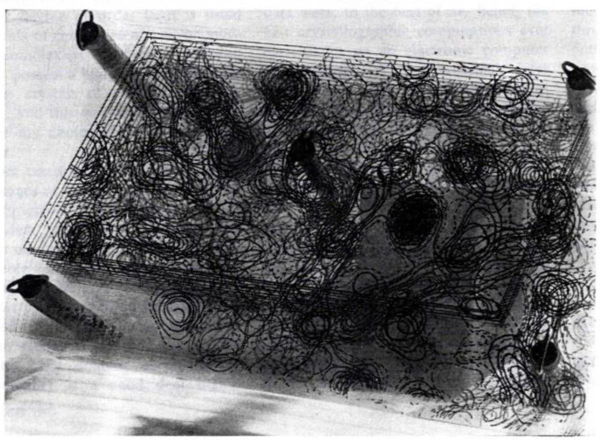
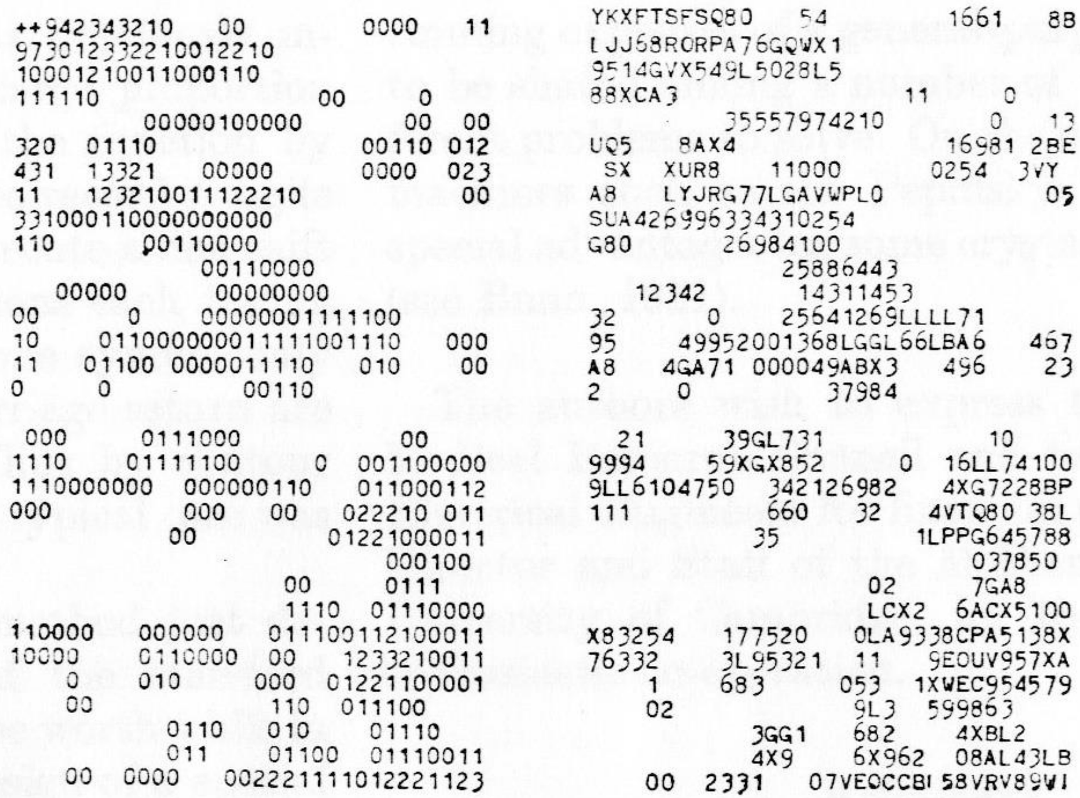


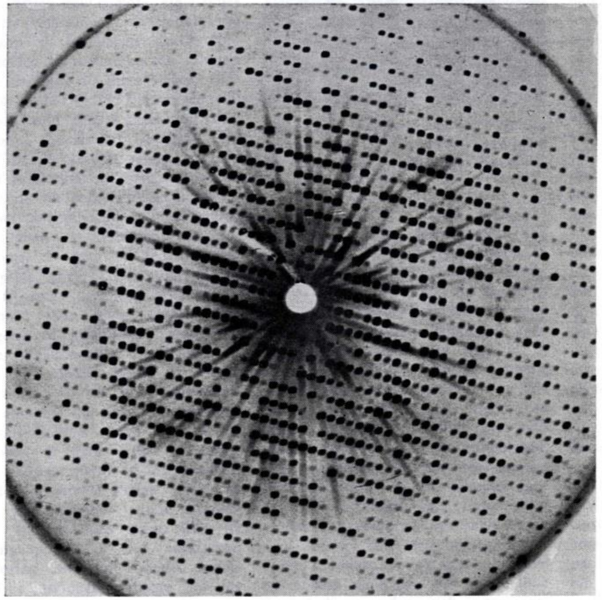
Fig. 2. Fourier synthesis of myoglobin at 6-Å resolution.



(a) (b)

Fig. 5. Patterson projection of whale myoglobin: 'contours' printed by the EDSAC. (a) Contours at intervals of 32. (b) Contours at intervals of 4.

Kendrew & Bennett: Elucidation of myoglobin to 6 Angstrom



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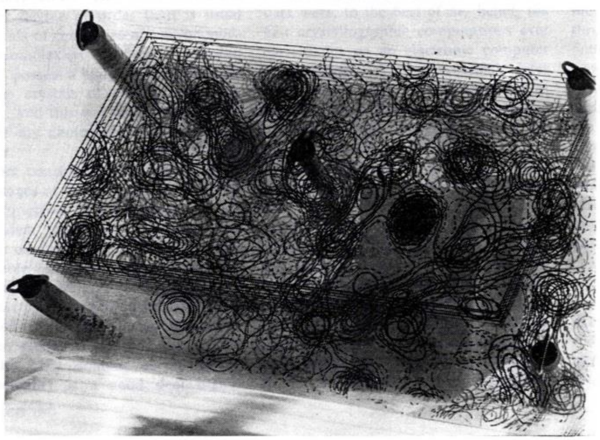

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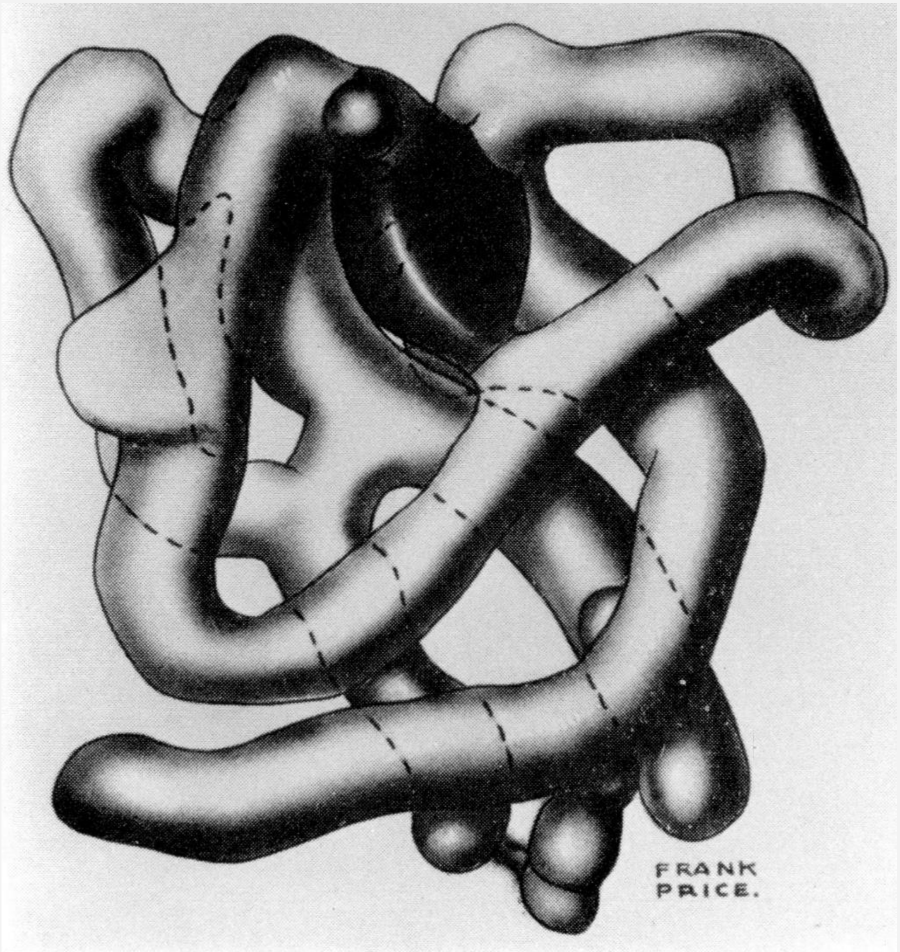
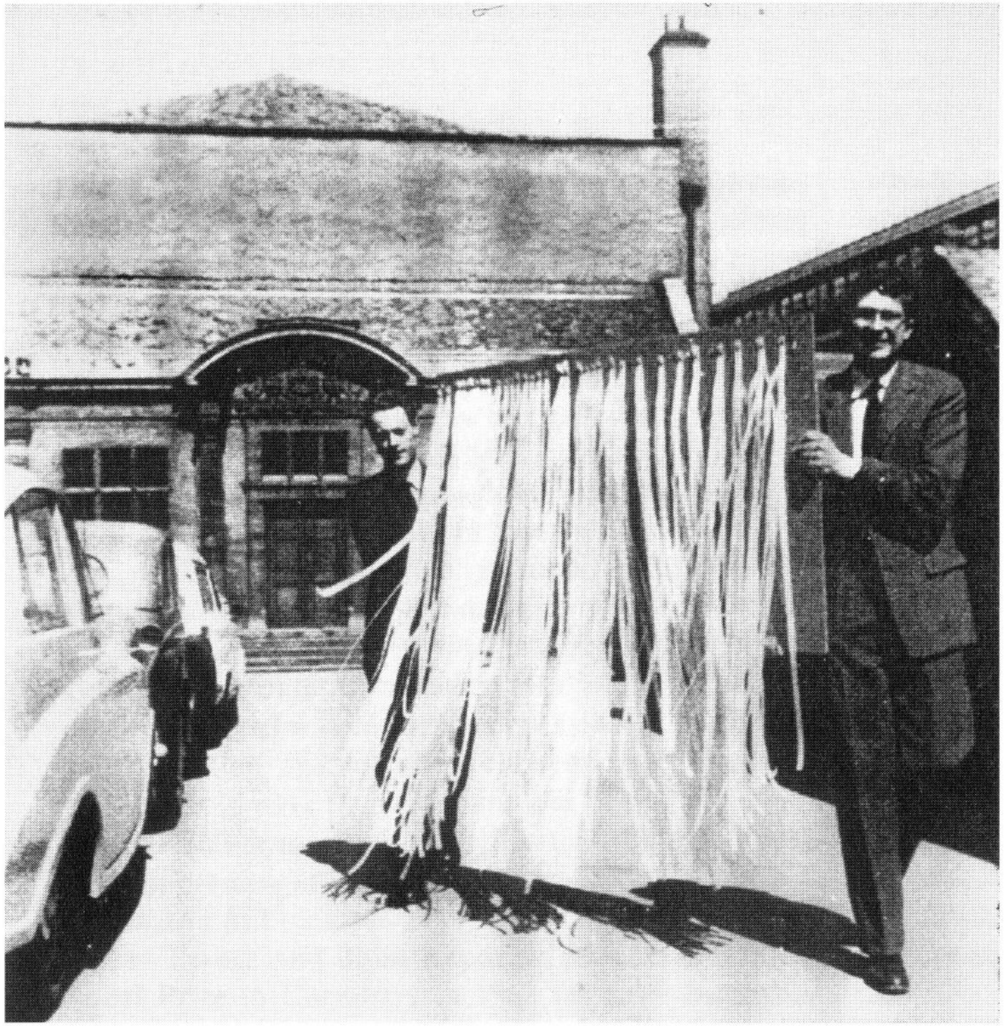


Kendrew & Bennett: Elucidation of myoglobin to 6 Angstrom



4.8 The hand-sorted data of the myoglobin calculations on EDSAC 2 are carried over from the Mathematical Laboratory to the MRC unit.

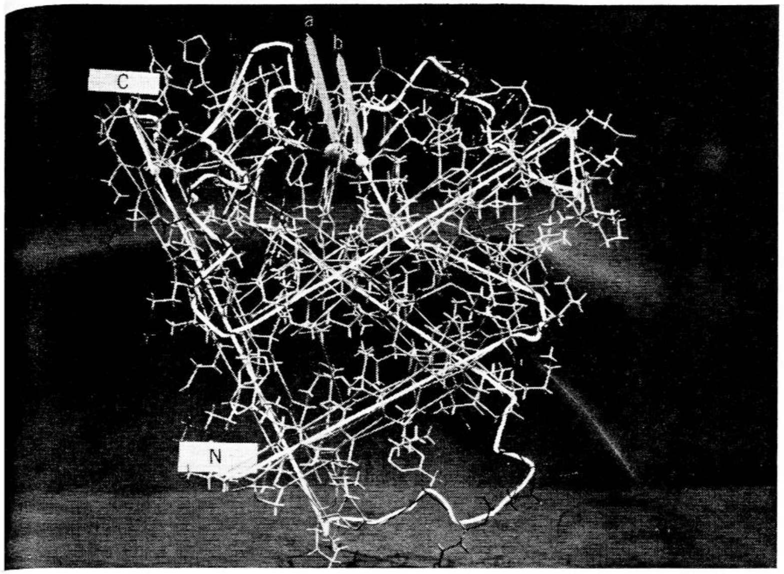


Fig. 7 (above). Model of the myoglobin molecule, derived from the 2-Å Fourier synthesis.

Kendrew & Bennett: Elucidation of myoglobin to 2 Angstrom, 1960